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Nonlinear Thermoelectricity : Cooling, Catastrophes and Carnot

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Plénière — GDR Physique Quantique Mésoscopique [Aussois Oct 2012]

SUMMARY

PART 0: What is *thermoelectricty* — and why care?

♠ good refrigerator *rarely* in linear-response regime

PART I: "Cooling & Catastrophes" R.W. arXiv:1208.6130

♠ *NON-LINEAR* scattering theory: fridge with *point-contacts*

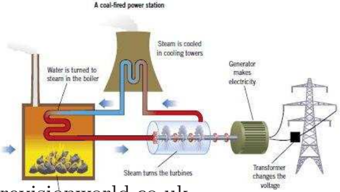
♠ RESULT = *catatstrophe aids cooling*
"thermoelectric quality" $\neq$ figure-of-merit

PART II: "Carnot and other Constraints" R.W. work in progress

♠ *Thermodynamic* & *quantum* constraints for refrigeration
in *"arbitrary"* quantum systems

WHAT is THERMOELECTRICITY?

Power-generation:
heat flow → electrical current



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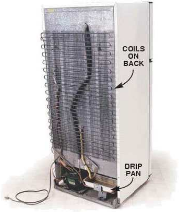
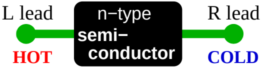
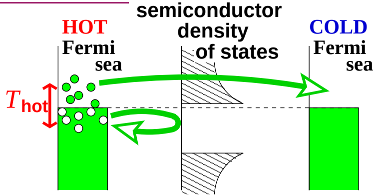
Refrigeration:
electrical current → heat flow



ENERGY-DEPENDENCE of TRANSMISSION

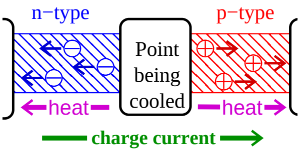


sub-Kelvin expt: semicond→supercond



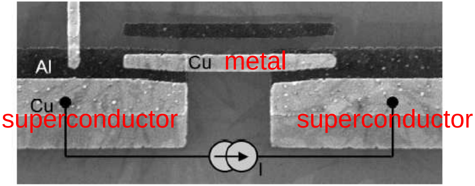
S-N-S THERMOELECTRIC COOLING

Rajauria, Luo, Fournier,Hekking, Courtois, and Pannetier (2007)

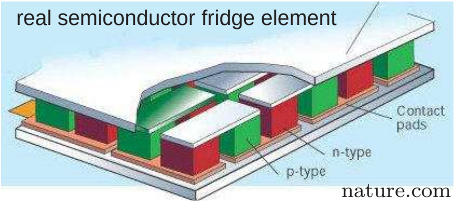


sub-Kelvin : S-N-S

Courtois and co−workers (2007−2009)



Refrigeration : 300mK $\rightarrow$ 100mK



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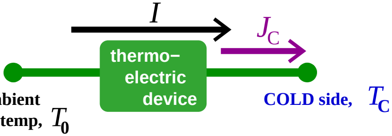
TEXT-BOOK THEORY for REFRIGERATION

minimal energy-conserving theory

$\Rightarrow$ *nearly*-linear response

e.g. Goldsmid (2009)
"Intro. to Thermoelectricity"

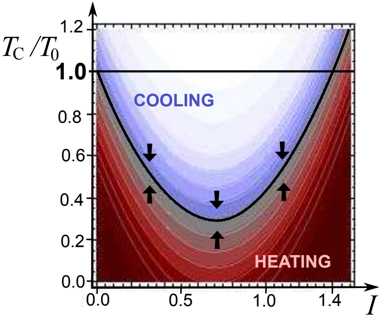


heat flow at cold: $J_C = \Theta\,(T_0 - T_C) \;-\; \underbrace{\Pi\, I} \;+\; \underbrace{\tfrac{1}{2} R I^2}$

Peltier effect (1834)

Joule heating (1840s)

- non-linear
- non-conserving



minimum: $T_C/T_0 = 1 - \tfrac{1}{2} Z T_0$

with $Z T_0 = \Pi^2/(R\Theta T_0)$

REFRIGERATION and ZT

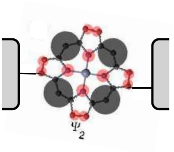
$$ZT = \frac{G\Pi^2}{(\Theta_{\rm electron} + \Theta_{\rm other})T}$$

Π is Peltier coefficient

G & Θ are electric & thermal conductance

♣ Currently: best bulk semiconductors $ZT \sim 1.5 - 2$

♣ Theory proposals: $ZT \sim 10$



Casati, Mejía-Monasterio, Prosen (2008)
Nozaki, Sevinçli, Li, Gutiérrez, Cuniberti (2010)
Saha, Markussen, Thygesen, Nikolić (2011)
Wierzbicki, Swirkowicz (2011)
Karlström, Linke, Karlström, Wacker (2011)
Gunst, Markussen, Jauho, Brandbyge (2011)
Rajput, Sharma (2011). Trocha, Barnaś (2012)

... but good fridges *rarely* in linear-response regime

♣ *Non-linear* theory for refrigeration with S-N-S

Rajauria, Gandit, Hekking, Pannetier, Courtois (2007).
Vasenko, Bezuglyi, Courtois, Hekking, (2009).

PART I

— COOLING & CATASTROPHES —

NON-LINEAR scattering theory

Fridge made out of *point-contacts*

SCATTERING THEORY BEYOND LINEAR RESPONSE

Linear response :

charge conductance Landauer & Büttiker (1957-1980s)
heat conductance Enquist & Anderson (1981)
thermoelectric Sivan & Imry (1986), Butcher (1990)

decoherence as "extra leads" Büttiker (1980s)

beyond linear response:

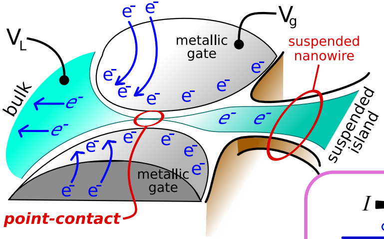
Hartree-like interactions included *self-consistently*

point-contact Moskalets (1995)
general Christen-Büttiker (1996)

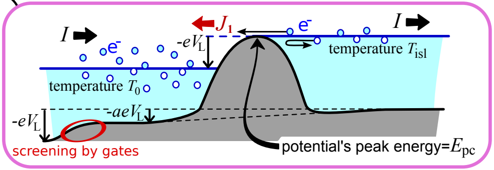
$$J_i = \int_{-\infty}^{\infty} \frac{\mathrm{d}\epsilon}{h} \, (\epsilon - eV_i) \, \mathcal{T}_{ij}(\epsilon) \, f\left[\frac{\epsilon - eV_j}{k_{\mathrm{B}} T_j}\right] \qquad \mathcal{T}_{ij}(\epsilon) = \mathrm{tr}\left[\mathcal{S}_{ij}^{\dagger} \mathcal{S}_{ij}\right]$$

Self-consistent loop: $\mathcal{S}_{ij}(\epsilon)$ ⇄ potential-distrib. in system

FRIDGE using POINT-CONTACTS at PINCH-OFF



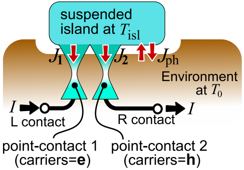
Expt: suspended structures
e.g. Heron, Fournier, Mingo, Bourgeois (2009)



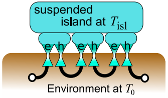
Long point-contact
only parameter = E_{pc}
(minimal tunnelling)

$\Rightarrow$ interactions
only modifies E_{pc}

Thermocouple

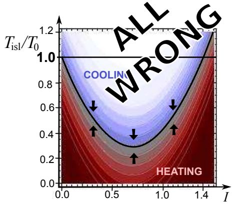


More complicated circuit

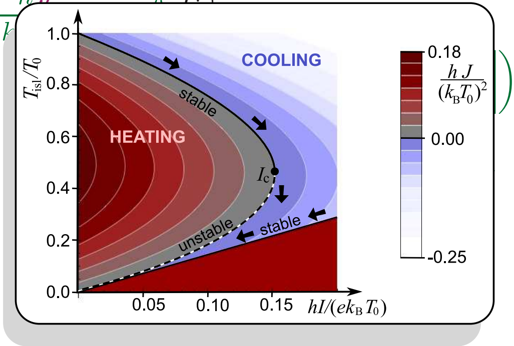


FRIDGE using POINT-CONTACTS at PINCH-OFF

nearly-linear "text-book" theory



Fully *non-linear* theory: Exact result.



Mathematically: "fold catastrophe" at I_C.

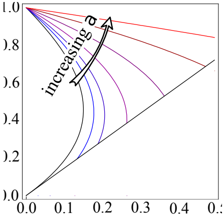
In principle: cooling to *absolute zero* (beyond catastrophe)

EFFECTS SUPPRESSING REFRIGERATION

Effect of phonons/photons

$$J_{\rm ph} = a(T_0^4 - T_{\rm isl}^4)$$

Stefan-Boltzmann Law



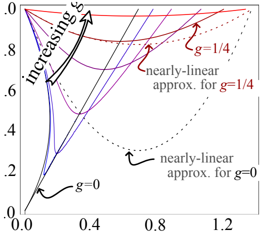
similar curves for T^2-photons

Pascal, Courtois, Hekking (2011)

Less strong thermoelectric effects

mimicked by point-contact

in parallel with resistance



Noting point-contact has $ZT = 1.4$

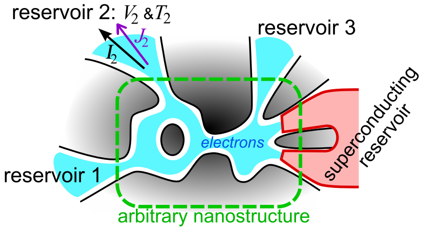
i.e. not a strong thermoelectric

Proposed nanodevices have $ZT \sim 10$

PART II

— CARNOT & OTHER FUNDAMENTAL CONSTRAINTS —

FUNDAMENTAL CONSTRAINTS on THERMOELECTRICITY



ZEROTH LAW ☑

EQUILIBRIUM

FIRST LAW ☑

$$\frac{\mathrm{d}}{\mathrm{d}t}E = \sum_i (J_i + V_i I_i) = 0$$

SECOND LAW : $\frac{\mathrm{d}}{\mathrm{d}t}S = \sum_i J_i/T_i \geq 0$

two-leads ☑

Bruneau, Jakšic, Pillet (2012)

many leads including superconductor ☑

QUANTUM CONSTRAINT : $J_i \;\geq\; -\dfrac{\pi^2}{6h}\, N_i\, (k_\mathrm{B} T_i)^2$

Stefan-Boltzmann for fermions: N° channels, $N_i \sim \frac{\text{lead cross-section}}{(\text{wavelength})^2}$

QUANTUM VS THERMODYNAMIC CONSTRAINTS

CONSTRAINTS on heat flow *out* of object being *refrigerated*

Carnot (1824)

Carnot efficiency: $$-J_{\mathrm{C}} \leq P_{\mathrm{supplied}} \frac{T_{\mathrm{C}}}{T_0 - T_{\mathrm{C}}}$$

Quantum: $$-J_{\mathrm{C}} \leq \frac{\pi^2}{6h} N_{\mathrm{C}} (k_{\mathrm{B}} T_{\mathrm{C}})^2$$

Ex. I: few channel nanostructure.

$P_{\mathrm{supplied}} \simeq 1\mathrm{pW}$
$T_{\mathrm{C}} \simeq 0.1\mathrm{K}$ & $T_0 \simeq 1\mathrm{K}$

Carnot $\Rightarrow$ $-J_c \leq 0.1\mathrm{pW}$
Quantum $\Rightarrow$ $-J_c \leq 0.01\mathrm{pW}$

Ex. II: kitchen freezer.

$P_{\mathrm{supplied}} \simeq 100\mathrm{W}$
$T_{\mathrm{C}} \simeq 260\mathrm{K}$ & $T_0 \simeq 300\mathrm{K}$
$N_{\mathrm{C}} \simeq 10^{10}$ per square-cm

Carnot $\Rightarrow$ $-J_c \leq 13\mathrm{W}$
Quantum $\Rightarrow$ $-J_c \leq 3.6\mathrm{W}$ per square-cm

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EXTRAS

Appendix : Calculation for FRIDGE using POINT-CONTACTS

Method. Scattering theory gives:

(a) non-linear heat current J in terms of temperature $T_{\rm isl}$ & voltage, V

(b) non-linear charge current I in term of temperature $T_{\rm isl}$ & voltage, V

Invert (b) & substitute for V in (a)

$\Rightarrow J$ in terms of temperature $T_{\rm isl}$ & $current\ I$.

Exact result.

$$\frac{h}{(k_{\rm B}T_0)^2}\, J = -\frac{\pi^2 T_{\rm isl}^2}{12 T_0^2} - {\rm Li}_2\left(1 - \exp\left[\frac{h(I_{\rm max}(T_{\rm isl}) - I)}{e k_{\rm B} T_0}\right]\right)$$

with $I \leq I^{\rm max}(T_{\rm isl}) = e k_{\rm B} T_{\rm isl} \ln[2]/h$