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NEUTRONS FOR SCIENCE

Poor qubits make for rich physics: noise-induced Zeno effect & noise-induced Berry phases.

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ICNF - Pisa, June 2009

Summary

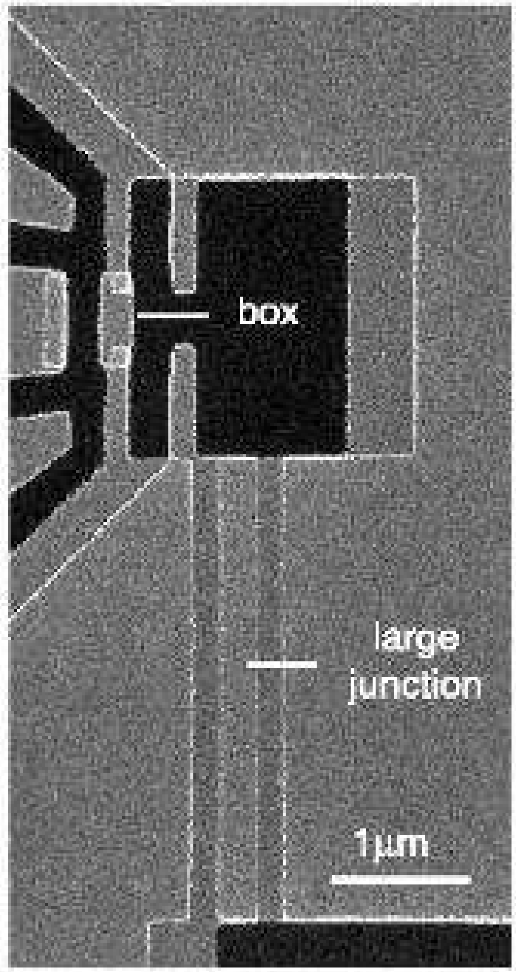
- ♣ Qubits in Markovian & non-Markovian noise

- ♢ Intro: Quantum Zeno paradox (no noise)
- ♣ Noise-induced Zeno paradox and "super"-Zeno paradox
- ♣ Experts: dangers of adiabatic renormalization group

- ♢ Intro: Berry phase (no noise)
- ♣ Noise-induced Berry phase

Qubits : fully controllable two-level systems

Saclay qubit: Vion et al (2002)



$$\mathcal{H} = B_x \hat{\sigma}_x + B_y \hat{\sigma}_y + B_z \hat{\sigma}_z$$

+ NOISE

or quantum environment

Noise making quantum physics richer?

♣ NO NOISE :

System characterized by wavefunction, $|\psi\rangle = u|\uparrow\rangle + v|\downarrow\rangle$

two-levels $\Rightarrow$ *TWO* independent variables

n-levels $\Rightarrow$ $(2n-2)$ independent variables

since $|u|^2+|v|^2=1$ & drop overall phase

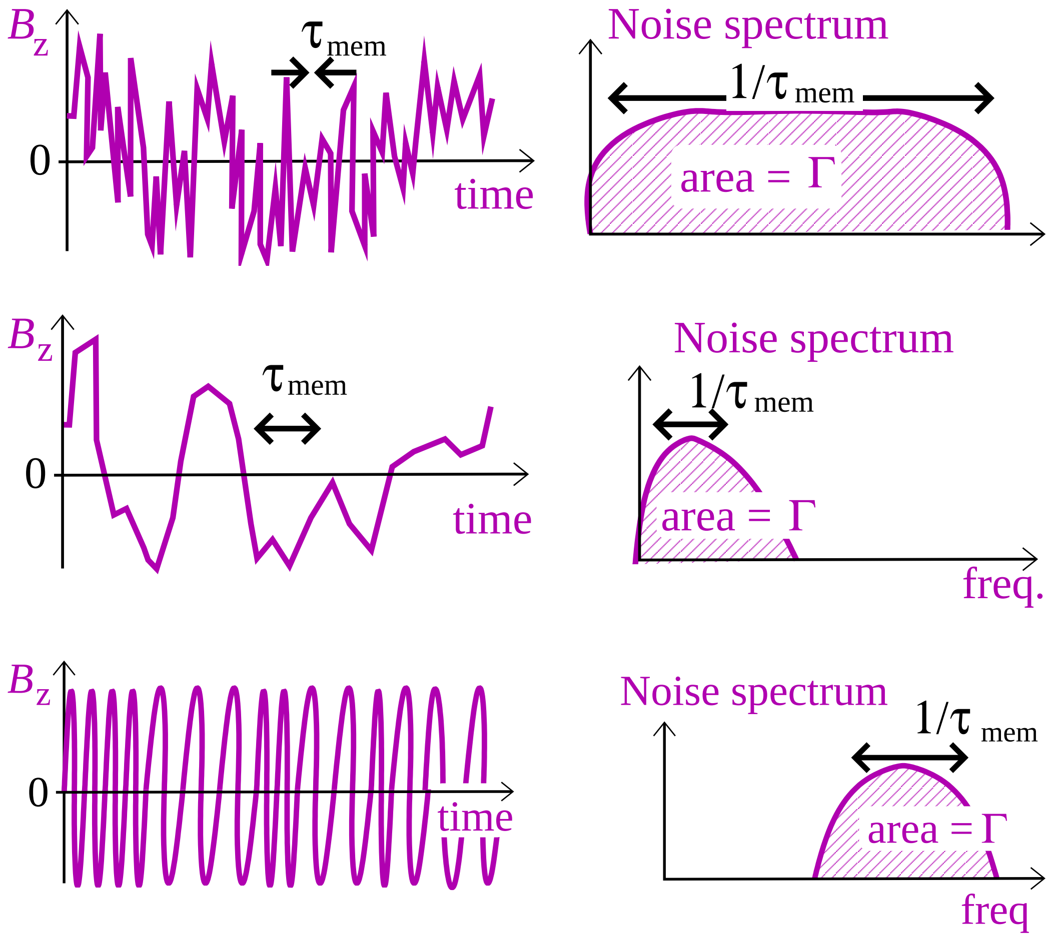
♣ WITH NOISE $\equiv$ WITH QUANTUM ENVIRONMENT

System characterized by density matrix, $\rho = \begin{pmatrix} a & b+\mathrm{i}c \\ b-\mathrm{i}c & 1-a \end{pmatrix}$

two-levels $\Rightarrow$ *THREE* independent variables

n-levels $\Rightarrow$ (n^2-1) independent variables

Markovian and non-Markovian noise



QUBIT (spin-half) +noise

$$\mathcal{H} = \mathcal{H}_{\text{qubit}} + \hat{\sigma}_z B_z(t)$$

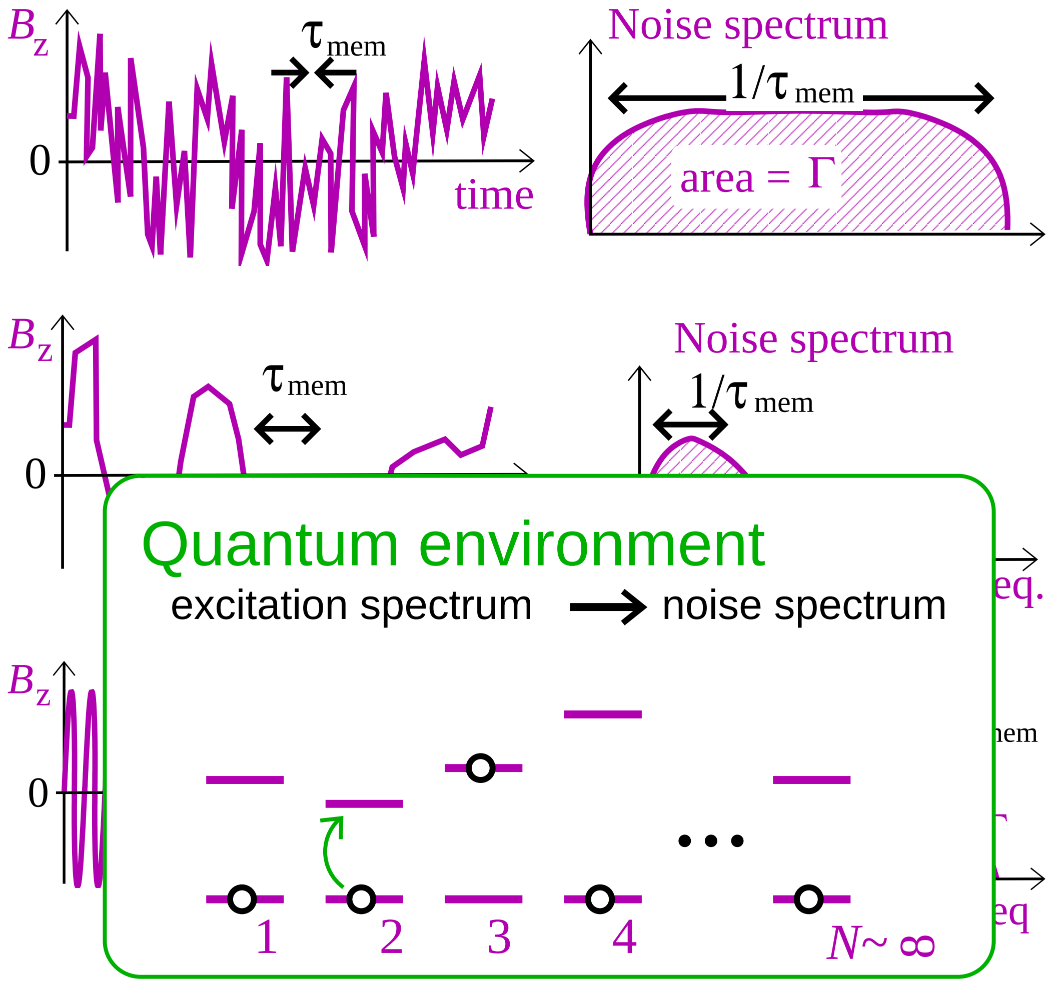
MARKOVIAN: $\Gamma\tau_{\text{mem}} \ll 1$

$\Rightarrow$ system weakly affected by noise on timescales τ_{mem}

NON-MARKOVIAN:

$$\Gamma\tau_{\text{mem}} \gg 1$$

Markovian and non-Markovian noise



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QUBIT (spin-half) +environ.

$$\mathcal{H} = \mathcal{H}_{\text{qubit}} + \hat{\sigma}_z K(\hat{a}^\dagger + \hat{a}) + \mathcal{H}_{\text{env}}$$

Quantum Zeno paradox

Misra- Sudarshan (1977)

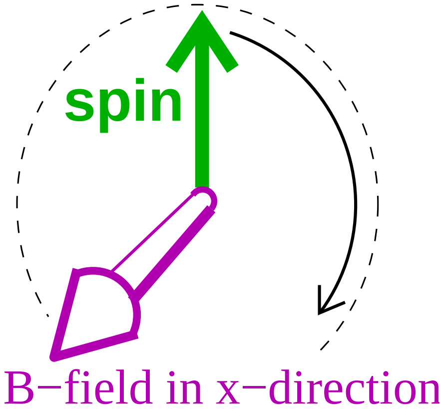
~~Zeno (400BC) :~~

~~paradoxes with summing infinite series,~~

~~fully resolved by Cauchy in 1800s~~

QUANTUM clock:

spin-half precessing

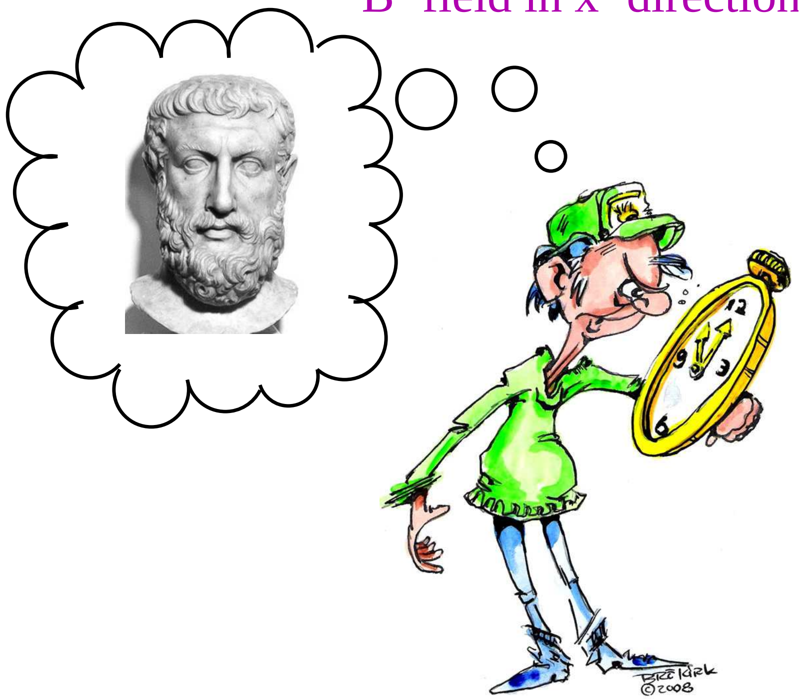


OBSERVATION: measure spin along z-axis

$$u|\uparrow\rangle + v|\downarrow\rangle \Longrightarrow \begin{cases} |\uparrow\rangle \text{ with prob.} = |u|^2 \\ |\uparrow\rangle \text{ with prob.} = |v|^2 \end{cases}$$

Observation at intervals $\tau_{\rm obs}$

$\longrightarrow$ spin-flip time $\sim 1/(B_x^2 \tau_{\rm obs})$



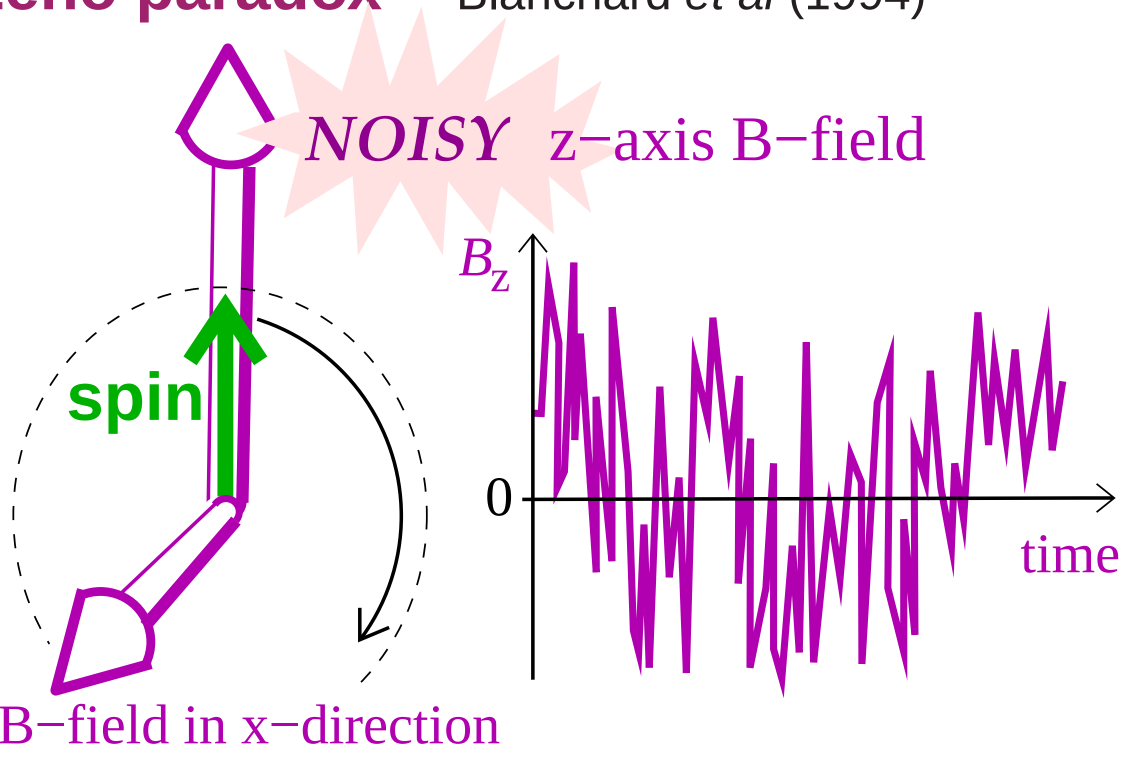
cf. no observations

$\longrightarrow$ spin-flip time $\sim 1/B_x$

Noise-induced Zeno paradox

Blanchard *et al* (1994)

White-noise
— strictly Markovian



Damped harmonic
oscillator eqn
for z-axis
spin polarization, s_z

underdamped $(B_x \gg \Gamma)$: $s_z \sim \exp[\mathrm{i}B_x t - \Gamma t]$

OVERdamped $(B_x \ll \Gamma)$: $s_z \sim \exp\left[-\frac{B_x^2 t}{\Gamma}\right]$ $\Rightarrow$ spin-flip time $\sim \dfrac{\Gamma}{B_x^2}$

— analogy to usual quantum Zeno with $\Gamma \sim 1/\tau_{\rm obs}$

Berry (1995)

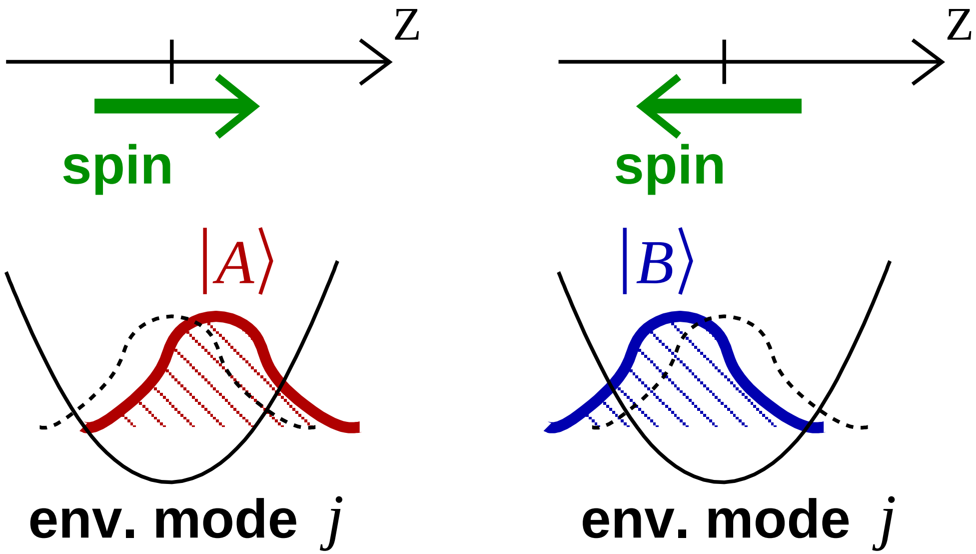
Adiabatic renormalization

Legget et al (1987)

QUBIT +environmnent : $\mathcal{H} = B_x \hat{\sigma}_x \; + \; \hat{\sigma}_z K(\hat{a}^\dagger + \hat{a}) + \mathcal{H}_{\rm env}$

qubit = SLOW

env modes = FAST



Born-Oppenheimer

no excited env modes

$$\langle j;A|j;B\rangle = 1 - K_j^2/\Omega_j^2$$

Franck-Condon

cf. Anderson orthogonality catastrophe

Overlap $= \prod_j \langle j;A|j;B\rangle = \exp\left[-\sum_j K_j^2/\Omega_j^2\right] \sim \exp[-\Gamma\tau_{\rm mem}]$

$$\hat{\sigma}_x \;\rightarrow\; \hat{\sigma}_x \times \exp[-F\,]$$

spin-flip time $\sim \dfrac{\exp[+\Gamma\tau_{\rm mem}]}{B_x}$

Exponentially slow → *"super"* Zeno effect

For experts : beyond Born-Oppenheimer

♣ ADIABATIC RENORMALIZATION ARGUMENT

qubit + N modes = qubit + highest mode + (N-1) modes

= renormalized qubit + (N-1) modes

Use Born-Oppenheimer mode-by-mode

requires $B_x \to 0$ *faster than* $\Omega_{\rm highest}$

...but do "irrelevant" terms flow $\to 0$*?*

♣ *POLARON MAPPING* [Mahan's book, Aslangul et al 86, Dekker 87]

Magic mapping: non-Markovian $\longrightarrow$ *Markovian* for most env

+ perturbation theory on new $\mathcal{H}$ (equiv to NIBA)

Terms missed by RG; in x,y-polarization but not *z-polarization.*

New rate $\sim \dfrac{B_x^2 B_z \tau_{\rm mem}^2}{(\Gamma\tau_{\rm mem})^{3/2}}$ *powerlaw in* Γ *not exponential*

Quick Intro to the Berry phase   Berry [1984]

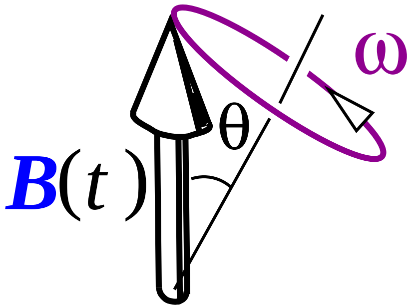
Observed in qubits Leek et al (2007)

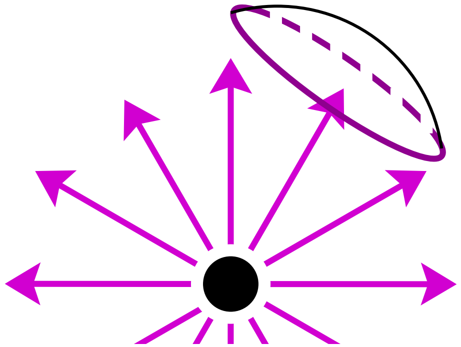
Proposed uses: metrology

Pekola et al (1999)

quantum computing

Jones et al (2000)





Rotate B-field

$\Phi_{\rm Berry}$ = solid-angle = monopole field $= 2\pi(1-\cos\theta)$

But Berry phase not alone:

$$\Phi_{\rm total} = \Phi_{\rm dyn} + \Phi_{\rm Berry} + \Phi^{(1)}_{\rm NA} + \Phi^{(2)}_{\rm NA} + \cdots$$

where $\Phi^{(\mu)}_{\rm NA} \sim (|B|t_{\rm p})^{-\mu}$

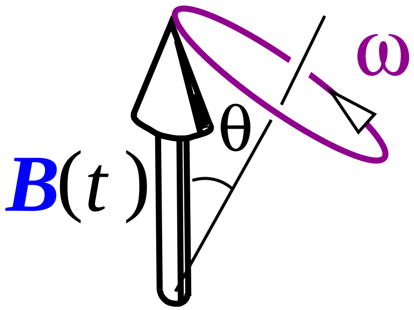
Measure $\Phi_{\rm Berry}$ with accuracy of 1 in 1000 $\rightarrow Et_{\rm p} \sim 10^{-3}$

Then $\Phi_{\rm dyn} \sim$ 1000 $\Phi_{\rm Berry}$

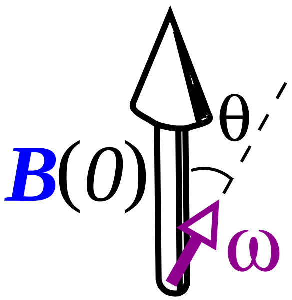
so must subtract $\Phi_{\rm dyn}$ with accuacy of 1 in 10^6

Rotating frame for Berry phase and non-adiab phases   Berry [1987]

Lab frame



Rotating frame



$$\Phi_{\mathrm{total}} = |\mathbf{B} + \omega| t_{\mathrm{p}} = B t_{\mathrm{p}} + \omega t_{\mathrm{p}} (1 - \cos\theta) + \mathcal{O}[\omega^2 t_{\mathrm{p}}]$$

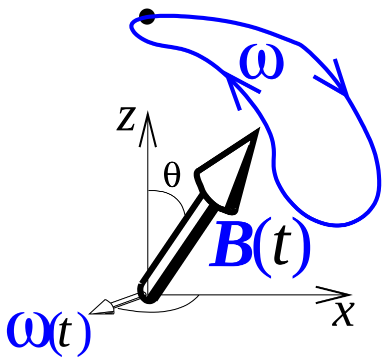
with $\omega t_{\mathrm{p}} = 2\pi$

Go from strongly time-dependent problem

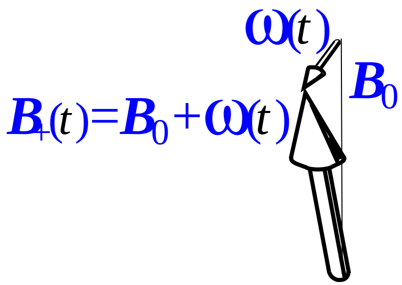
to (almost) time-independent problem

General transformation:

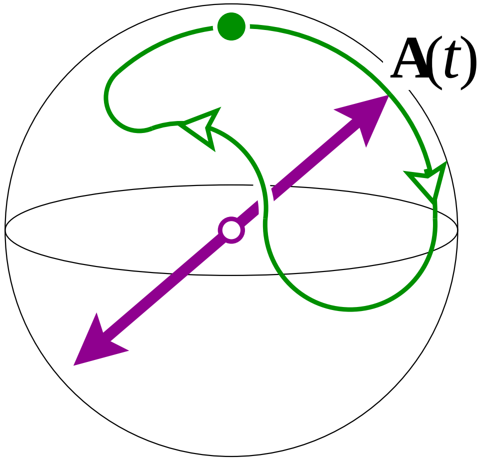
$$\mathcal{H}_{\mathrm{rot}} = \mathrm{i}[\mathrm{d}[\mathcal{U}/\mathrm{d}t]\mathcal{U}^{-1} + \mathcal{U}\mathcal{H}_{\mathrm{lab}}\mathcal{U}^{-1}$$



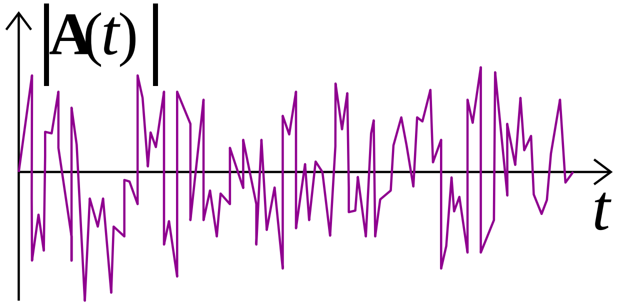
$\Rightarrow$



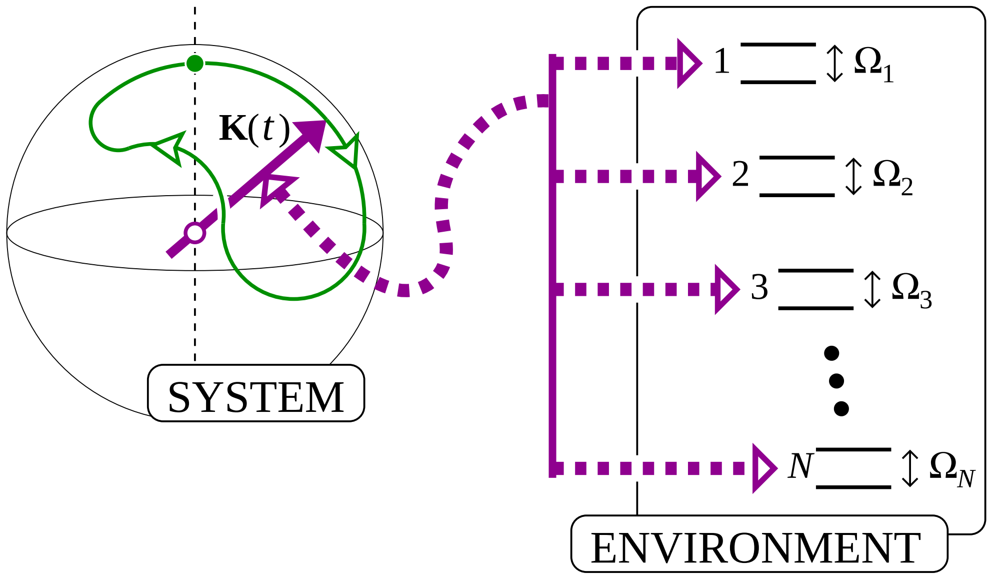
Making Berry phases with noise alone?



$$\mathbf{A}(t) = \mathbf{e}(t) \sum_{i=1}^{N} A_i \cos(\Omega_i t + \phi_i)$$



Quantum equivalent
(harder to realize)



Noise-induced Berry phase

♣ Rotate MARKOVIAN environment:

Carollo et al (2006) Dasgupta-Lidar (2007) Syzranov-Makhlin (2008)

Total phase as for conventional Berry phase
+ decoherence time $T_2 \ll t_{\rm P}$

tricks to keep phase (decoherence-free subspace)

♣ Rotate NON-MARKOVIAN environment:

$\Phi_{\rm total} = \Phi_{\rm Berry} + \Phi^{(2)}_{\rm NA} + \cdots$ with $\Phi^{(2)}_{\rm NA} \sim 1/(\Gamma^{3/2} \tau_{\rm mem}^{-1/2} t_{\rm p}^2)$

Measure $\Phi_{\rm Berry}$ to 1 in 1000 $\rightarrow E_2 t_{\rm p} \sim 10^{-3/2} \simeq$ 31

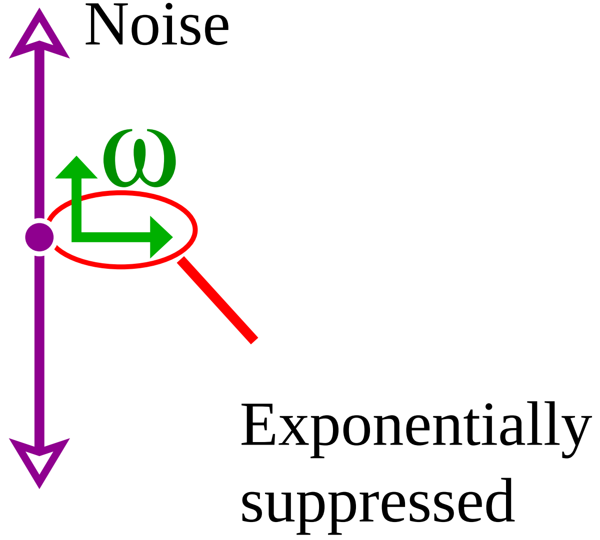
Also decoherence not significant for $\Gamma t_{\rm p} > 1$

less decoherence in long expt!

Why is noise-induced Berry phase like this?

Why a Berry phase?

Rotating frame:
“super”-Zeno effect suppresses $\omega_\perp$
$\Rightarrow$ $\omega_{||}$ gives $\Phi_{\rm BP}$



Why so little decoherence?

Magic of non-Markovian env.
No decoherence without transverse field

Why so little non-adiabaticity?

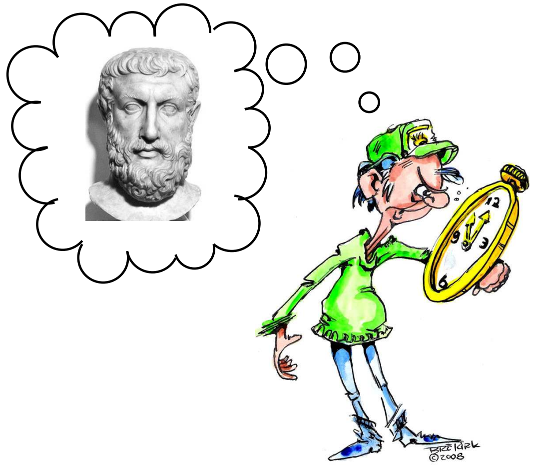
Not sure yet!

Conclusions

◇ Rich quantum physics with non-Markovian noise

◇ orthogonality catastrophe

or "super" Zeno effect



♣ non-Markovian noise makes a "better" Berry phase

get rid of unwanted Φ_{dyn} or $\Phi^{(1)}_{\mathrm{NA}}$

♣ What other new things can noise do?

♣ Don't slip up