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Les Houches, Fevrier 2010

Quantum revivals — connections to bouncing neutrons?

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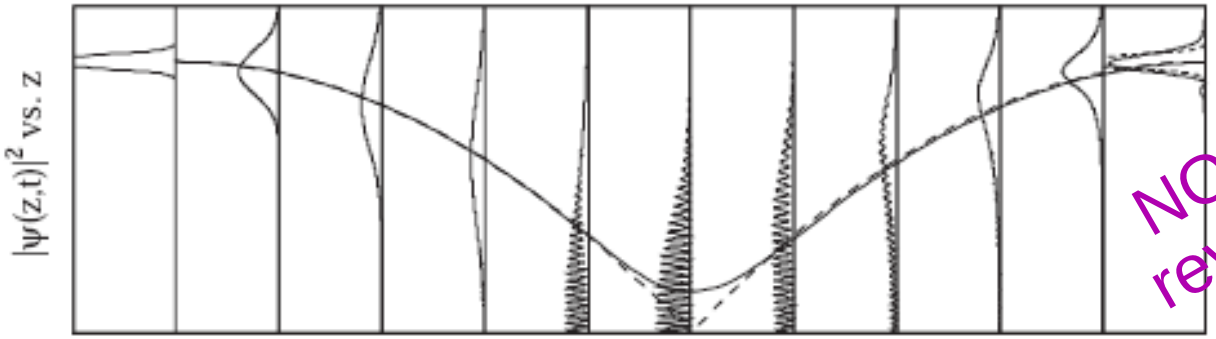
REVIEWING A REVIEW:

R.W. Robinett, *"Quantum wavepacket revivals"*, Physics Reports **392**, 1-119 (2004)

What is a “quantum revival”?

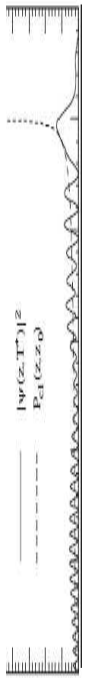
Quantum revival : wavepacket spreads out and then

... *unexpectedly* recombines

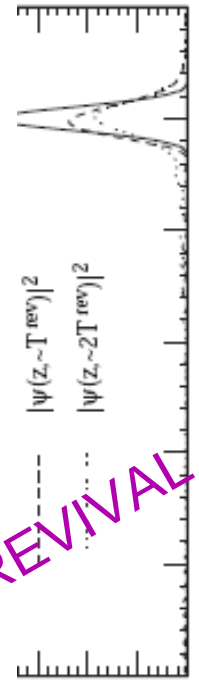


NOT revival

after many periods



after ~160 periods



REVIVAL

quantum version of Poincaré’s return??

Outline

♣ Two types of revival problem.

[a] initial-state localized in position

[b] initial-state localized in energy

♣ Simple picture of neutron expts $\Rightarrow$ localized in energy

♢ Theory of revivals

♢ Mathematical dangers!

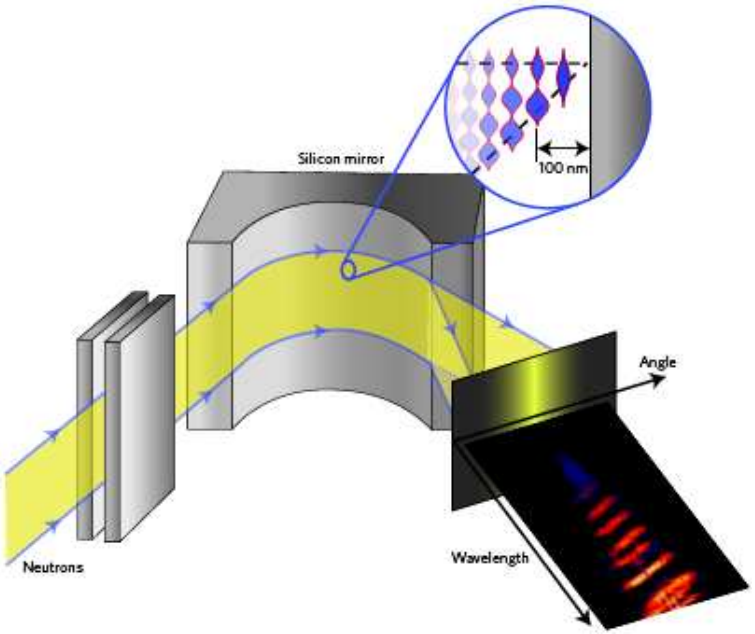
♣ Revivals in Neutron experiment

- ***Observable??*** YES

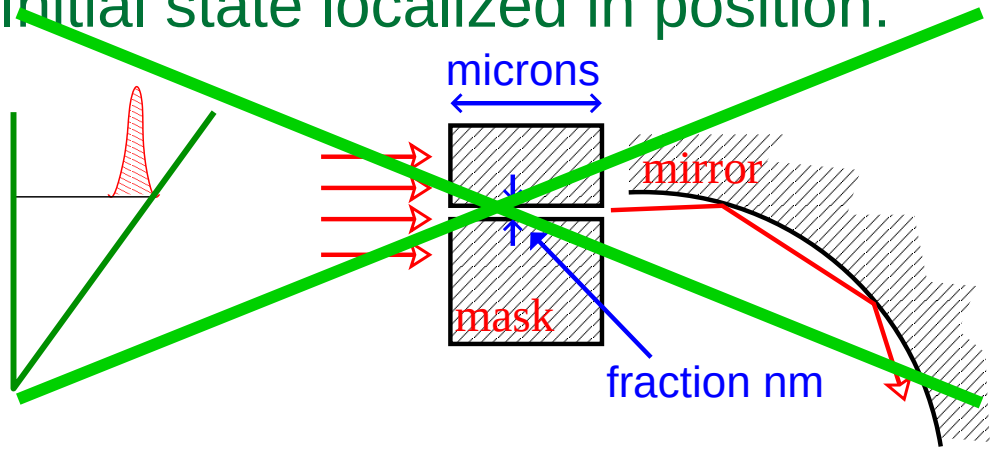
  ...but must take theory to lower levels + finite lifetimes

- ***Useful tool??*** NOT SURE

Neutron expt: initial-state localized in position or energy

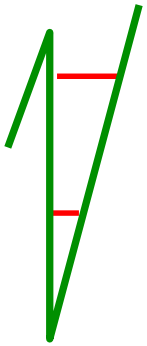


Initial state localized in position:



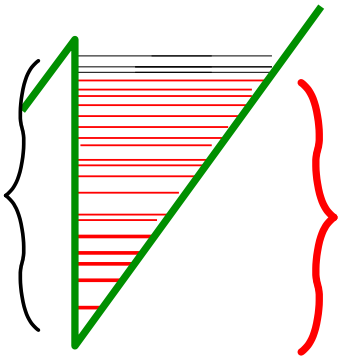
Initial state localized in *energy*:

HIGH energy



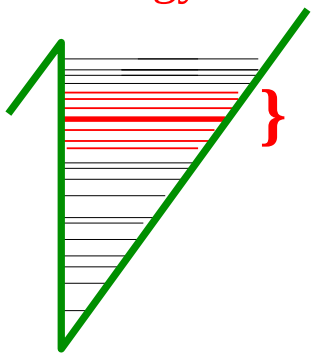
LOWER energy

approx
50 levels

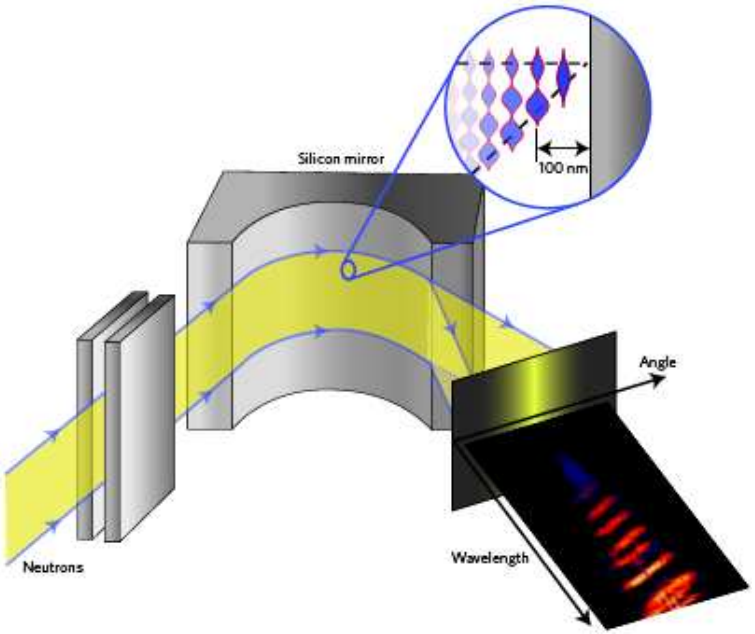


occupied

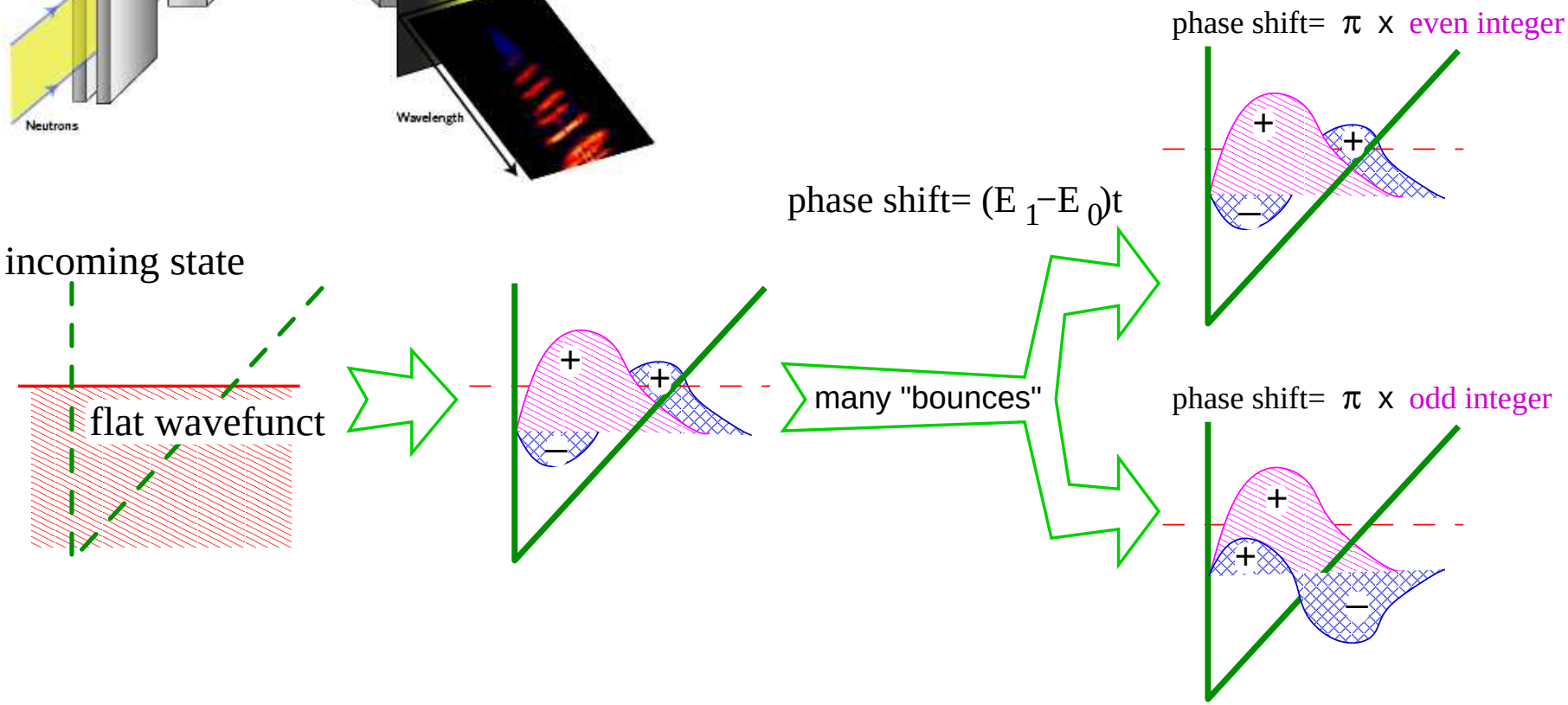
LOWER energy
+ energy filtering



Neutron expt: final state



Simplest case: only 2 levels

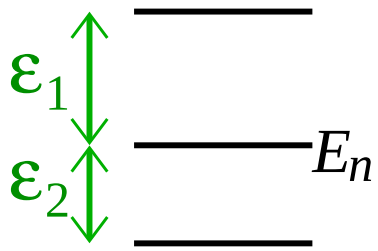


Revivals for few levels

not in Robinett's review — he only considers many-levels

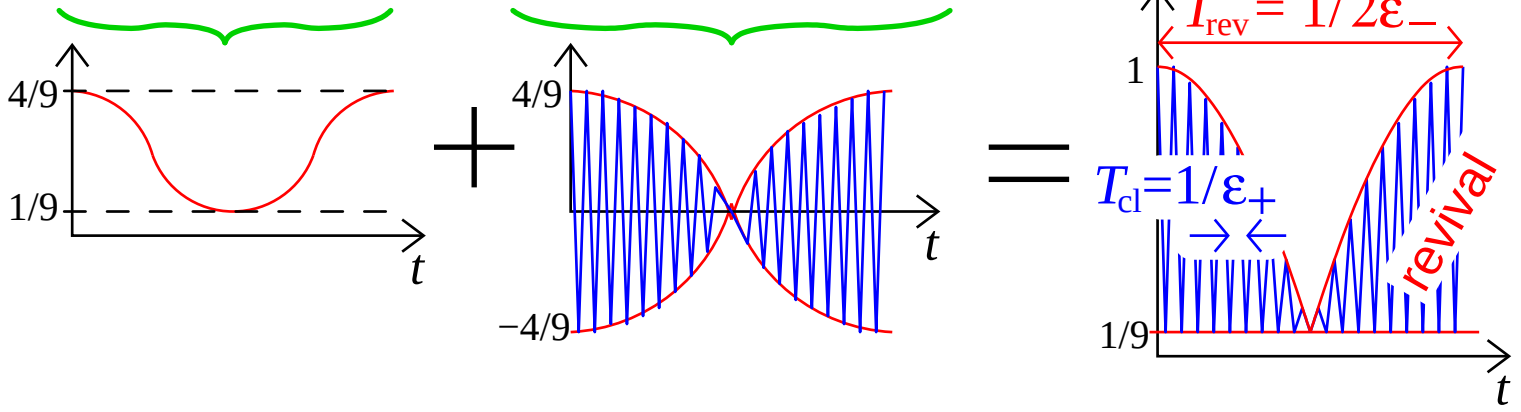
Overlap initial and final wavefunction:

$$A(t) = \langle \psi_0 | \psi(t) \rangle$$



$$|A(t)|^2 = \tfrac{1}{9}\left|1 + \mathrm{e}^{\mathrm{i}\epsilon_1 t} + \mathrm{e}^{\mathrm{i}\epsilon_2 t}\right|^2$$

$$= \tfrac{1}{9}\left(1 + 4\cos^2 \epsilon_- t + 4\cos \epsilon_- t \cos \epsilon_+ t\right)$$



$\epsilon_+ = \tfrac{1}{2}(\epsilon_1 + \epsilon_2) \Rightarrow$ classical oscillation period

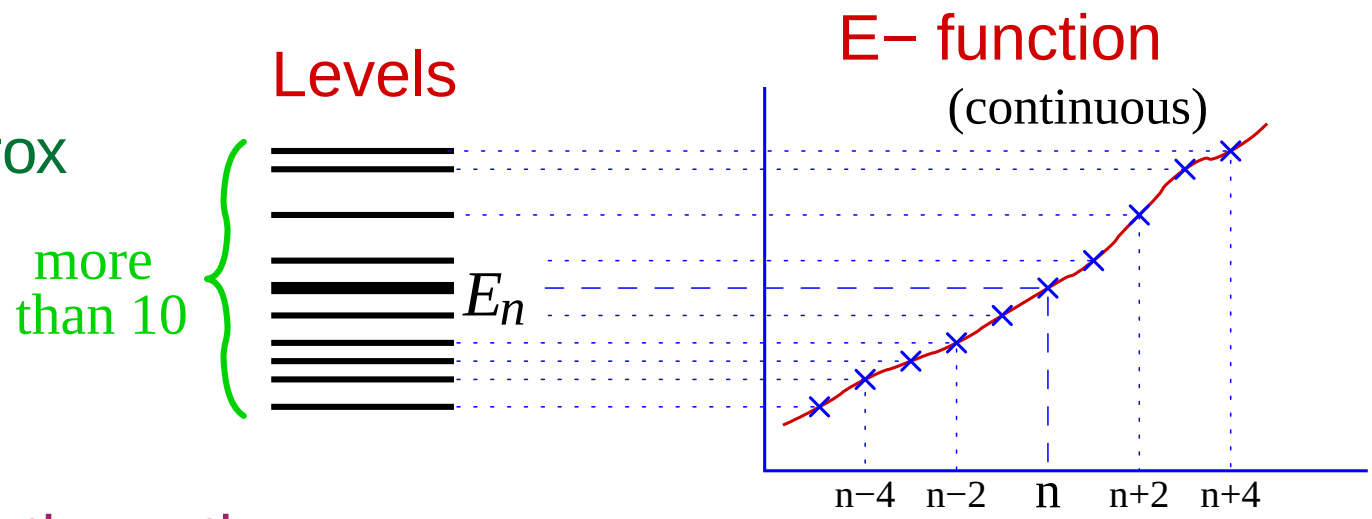
cf. oscillator where freq = level-spacing

$\epsilon_- = \tfrac{1}{2}(\epsilon_1 - \epsilon_2) \Rightarrow$ REVIVAL period

for oscillator $\epsilon_- = 0$

Revivals for many levels

Continuum approx



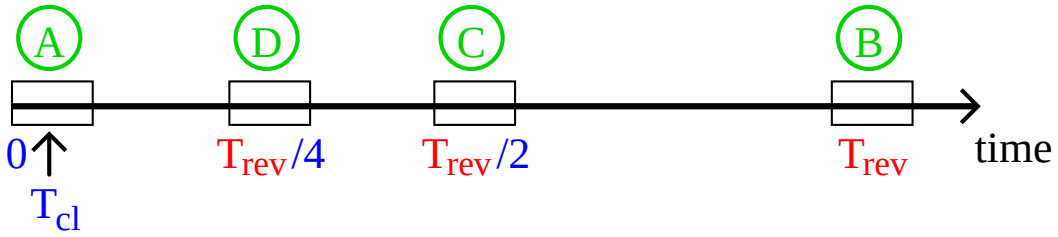
Lots of pretty mathematics

1st dervative $\Rightarrow$ classical oscillations $T_{\rm cl}$

2nd derivative $\Rightarrow$ REVIVALS with period $T_{\rm rev} \gg T_{\rm cl}$

analogy: 2-slit interference $\longrightarrow$ grating interference

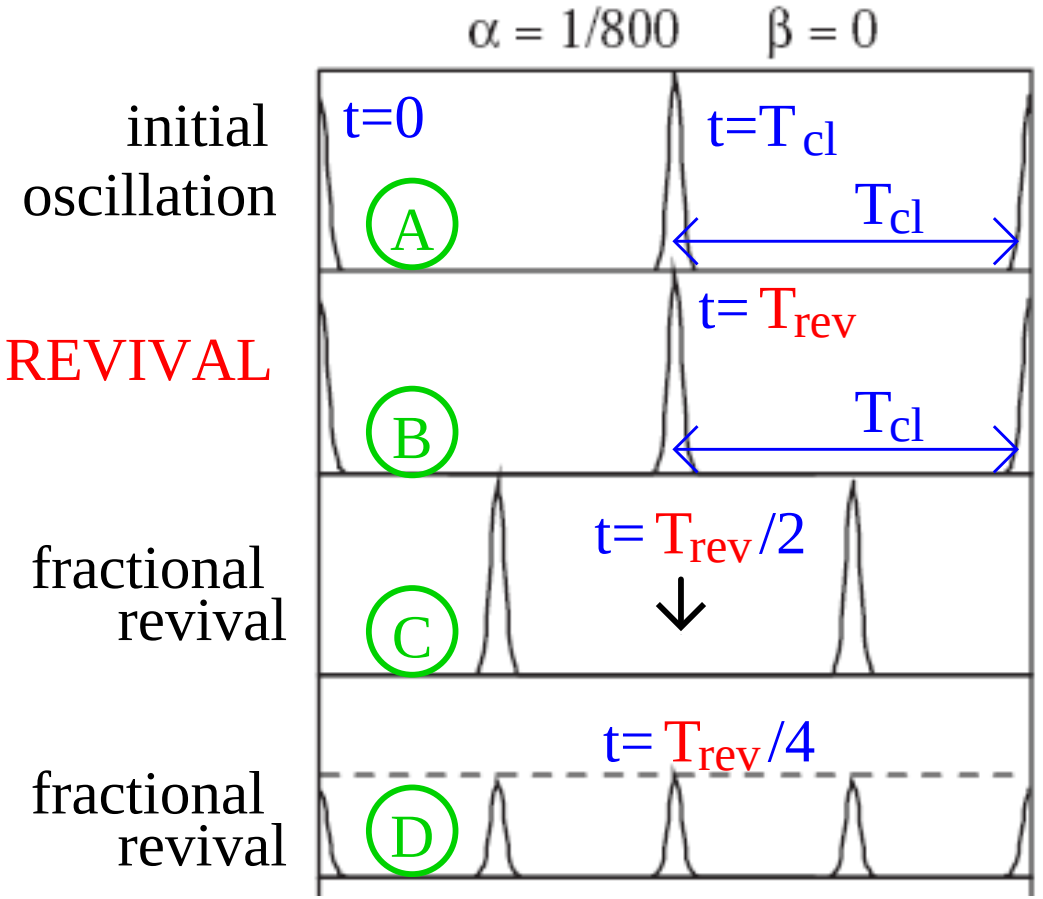
Revivals and fractional revivals



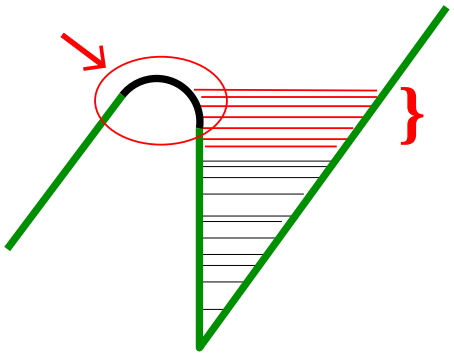
fractional revivals

in continuum theory

not in 3-level model



Mathematical dangers



[1] For neutrons — only 50 levels above mirror

use N levels in initial-state for $N = 10$

Is continuum model ***good enough***?

- ♢ Errors go like $1/N$ or not?
- ♢ Can we do better?

[2] We know:

Hamiltonian ⇒ spectrum ⇒ E-function ⇒ ***Revivals***

...but what about

Revivals ⇒ E-function ⇒ spectrum ⇒ ***Hamiltonian***

Revivals reveal neutron-mirror interaction?? i.e. trapping potential

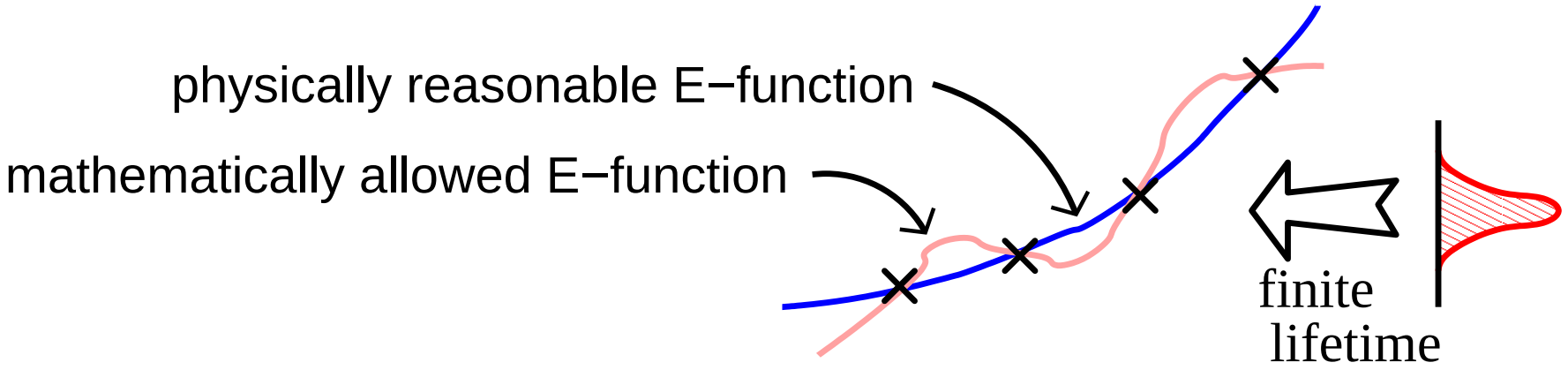
Danger: Difficult & uniqueness unknown

dependence on expt uncertainties??

cf. hearing sound of drum (2D — non-unique)

Mathematical dangers

[3] No unique definition of E-functions??

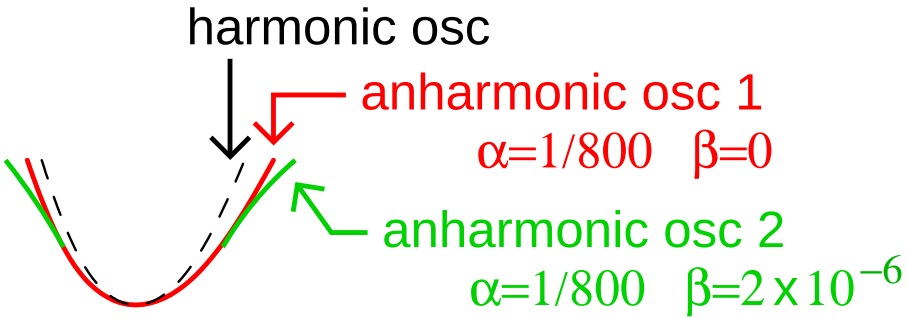


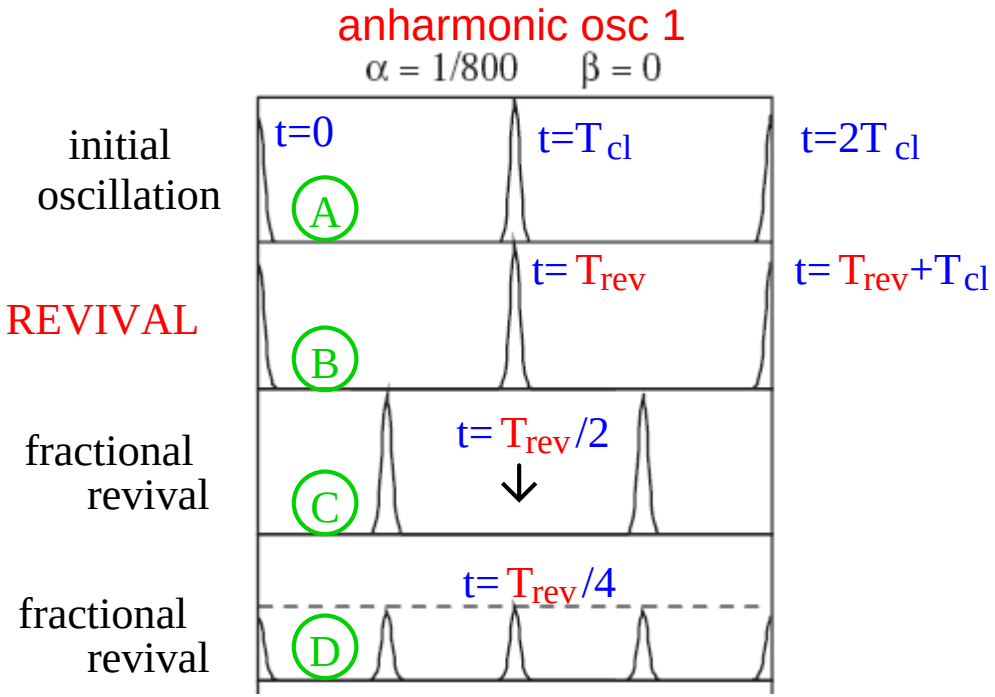
♢ Is this a ***formal problem*** or a ***practical one***?

♢ Will uncertainity introduced by life-times make it ***worse***?

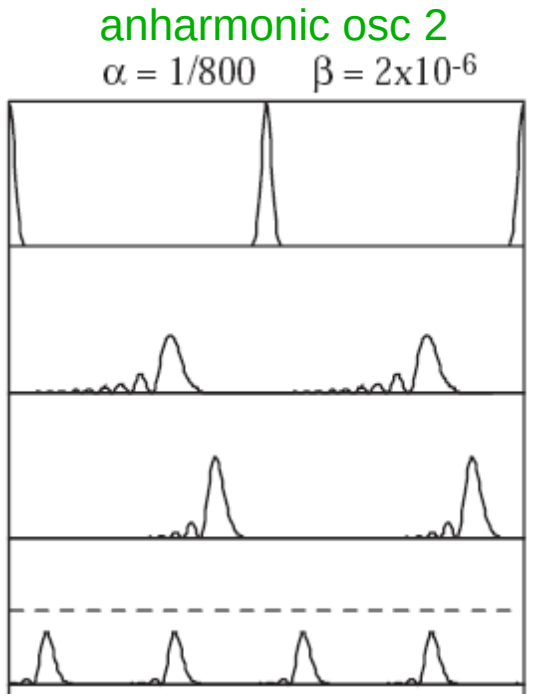
Revivals as useful tool??

Very small changes
in spectrum





$T_{\rm rev} \sim 800 T_{\rm cl}$



$T_{\rm rev} \sim 2933 T_{\rm cl}$

Exciting or Frightening??

Summary

♣ Two types of revival problem:

[a] initial-state localized in position

[b] initial-state localized in energy $\Leftarrow$ easiest for neutron expt

♢ Theory: Robinett, Physics Reports **392**, 1-119 (2004)

... ***but*** helpful picture is beating in 3-level model

♣ Revivals in Neutron experiment

- ***Observable??*** YES
- ***Useful tool??*** NOT SURE

extremely sensitive to expt parameters

♣ NEED theory for intermediate N° of levels & finite lifetimes