INSTITUTE LAUE LANGEVIN
GRENOBLE

NEUTRONS FOR SCIENCE

The rivalry between *interference* and *tunnelling* in chaotic quantum-dots

Robert S. Whitney

♣ M. Macucci & R.W. *work in progress.*
♣ R.W. Phys. Rev. B **75**, 235404 (2007)

♣ C. Petitjean, Ph.Jacquod & R.W. JETP Lett. **86**, 736 (2007) and Phys. Rev. B *to be published* (2008)
♣ R.W. & Ph. Jacquod, PRL **96**, 206804 (2006)
♣ Ph. Jacquod & R.W., PRB **73**, 195115 (2006)
♣ R.W. & Ph. Jacquod, PRL **94**, 116801 (2005)
See also: Brouwer & co-workers and Haake & co-workers.

Orsay - Decembre 2007

QUANTUM CHAOS : quantum mechanics of a classically chaotic system.

Individual systems: unique
... but average properties: UNIVERSAL

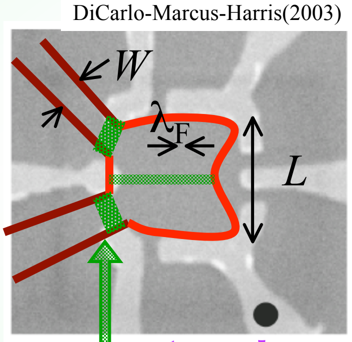
Transport properties through open nanoscale chaotic system:

$$\lambda_F << \rho W << L$$

Good conductor (no Coulomb blockade)
Semiclassical: $\lambda_F << W$

***Lots* of chaos**

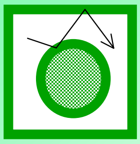
DiCarlo-Marcus-Harris(2003)



tunnel barriers
transparency, ρ

❶ **Tunnel-barriers *suppress* weak-localization** (interference)
❷ ***Interplay* of quantum-noise and tunnelling**
❸ **Massive conductance enhancement – interference *suppresses* tunnel-barriers**

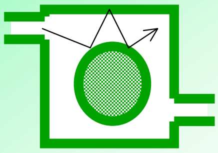
Regimes in *closed* and *open* systems



Level-statistics, etc

system specfic	Random matrix	

10 — $\frac{\text{system size}}{\text{wavelength}}$



Conductance, etc

system specfic	Random matrix	CLASSICAL regime

10 — $\frac{\text{system size}}{\text{wavelength}}$

Measure of classicalness: Ehrenfest time/dwell time

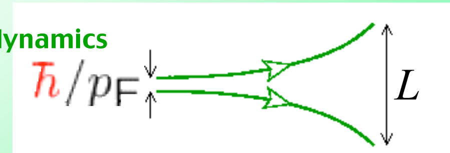
Aleiner-Larkin (1996)

Ehrenfest time, τ_E = time for wavepacket to spread to classical scale

Ehrenfest theorum: spread follows classical dynamics

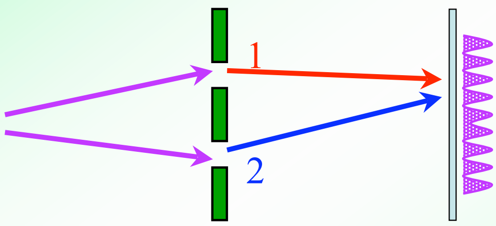
$$\tau_E = \Lambda^{-1} \ln [L/\lambda_F]$$

Lyapunov exponent



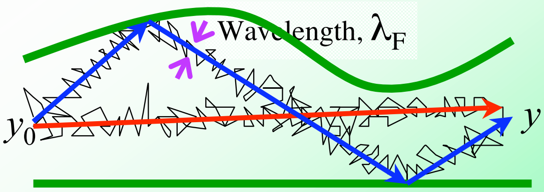
Semiclassics : *ray optics* for the 21st century

Ray optics:
Young's slits



$$\langle |A_1 e^{iS_1/h} + A_2 e^{iS_2/h}|^2 \rangle = \langle A_1^2 \rangle + \langle A_2^2 \rangle + \langle A_1 A_2 \cos[(S_1 - S_2)/h] \rangle$$

Semiclassics:
Feynman path integral (saddle-point)
Van Vleck (1920s), Gutzwiller (1970s)

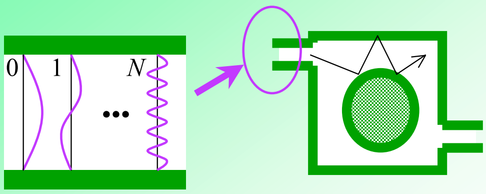


Probability

$$\sim \sum_{\gamma 1} \sum_{\gamma 2} A_{\gamma 1} A_{\gamma 2} \exp[i(S_{\gamma 1} - S_{\gamma 2})/h] \quad \text{becomes} \quad \sum_{\gamma 1} A_{\gamma 1}^2$$

Berry "diagonal approximation" (1985)

Semiclassics : *ray optics* for the 21st century



scattering matrix:

$$\mathrm{S} = \begin{pmatrix} \mathrm{r} & \mathrm{t}^{\dagger} \\ \mathrm{t} & \mathrm{r}' \end{pmatrix}$$

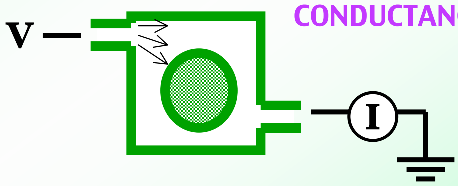
Baranger *et al* (1993)

$$\mathbf{t}_{nm} \sim \mathcal{A} \sum_{\gamma} A_{\gamma} \exp[\mathrm{i}\, S_{\gamma} / h]$$

Coupling to lead mode wavefunctions

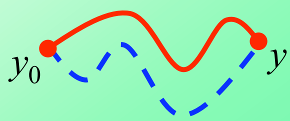
$$\mathcal{A} \sim \int \mathrm{d}y_0 \int \mathrm{d}y \; \langle n|y\rangle \langle y_0|m\rangle$$

Landauer–Buttiker



CONDUCTANCE : $G = \mathrm{I/V} = (\mathrm{e}^2/\mathrm{h})\ \mathrm{tr}[\mathbf{t}^{\dagger}\mathbf{t}]$

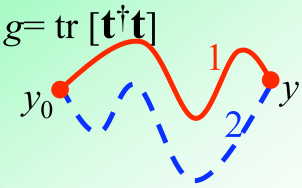
$g = \mathrm{tr}\ [\mathbf{t}^{\dagger}\mathbf{t}]$



AVERAGE transport properties

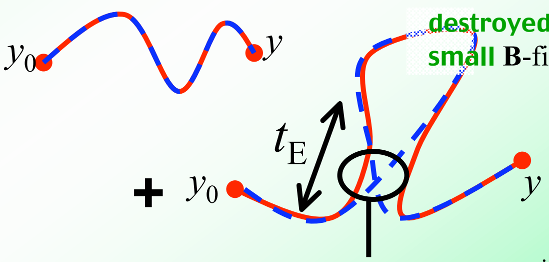
$$\left\langle \left| A_1\, e^{\mathrm{i}S_1/h} + A_2\, e^{\mathrm{i}S_2/h} \right|^2 \right\rangle = \left\langle A_1^{\,2} \right\rangle + \left\langle A_2^{\,2} \right\rangle + \left\langle A_1 A_2 \cos[(S_1 - S_2)/h] \right\rangle$$

$g = \mathrm{tr}\ [\mathbf{t}^{\dagger}\mathbf{t}]$

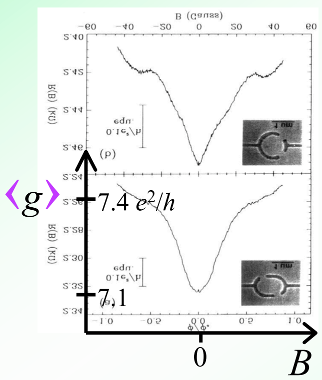


average

$\langle g \rangle = \langle \mathrm{tr}[\mathbf{t}^{\dagger}\mathbf{t}] \rangle$



typical phase diff. $= \pi \rightarrow e^{\mathrm{i}\pi} = -1$

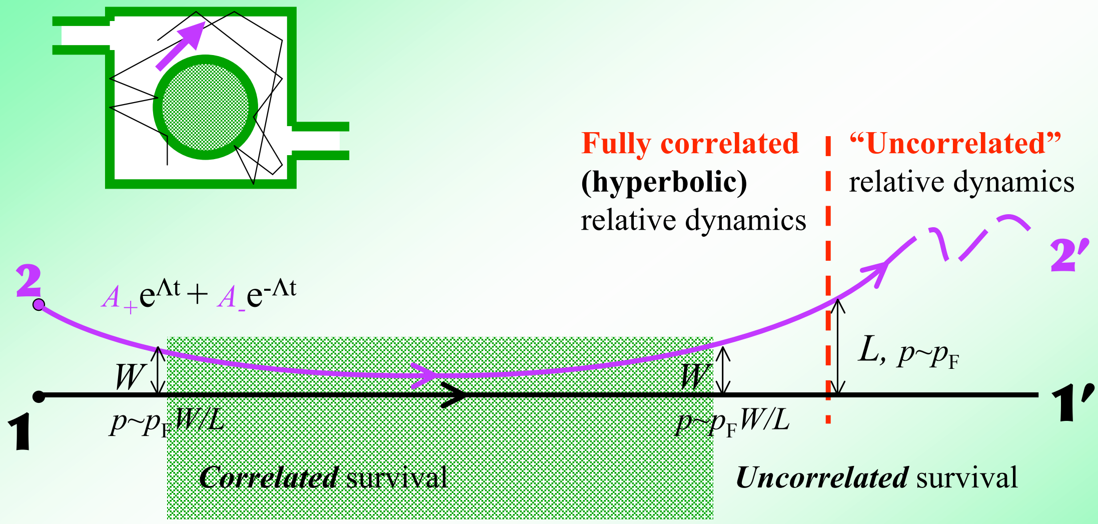


Chang *et al*, PRL **73**, 2111 (1994)

$$\langle g \rangle_{\mathrm{RMT}} = N/2 \quad - 1/4$$

Weak-loc: Richter-Sieber (2002)

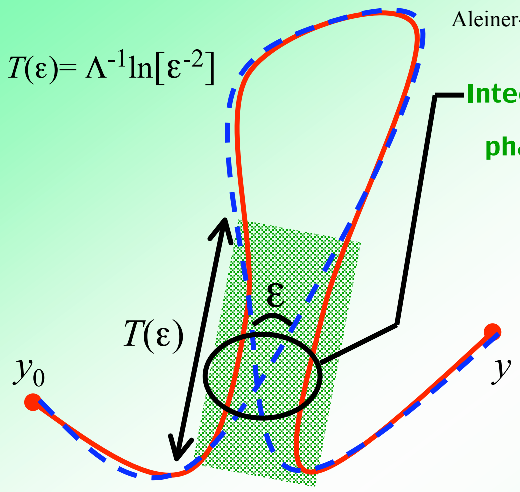
Modelling "*generic*" classical chaos



also classical paths 1 → 2′ and 2 → 1′

Weak-localization in classical limit

Aleiner-Larkin (1996), Rahav-Brouwer (2005), Jacquod-Whitney (2006)



Integrate over ε : dominated by $\varepsilon \approx (\lambda_F/L)^{1/2}$

phase diff. $\Phi = E_F \varepsilon^2 / (\Lambda \hbar)$

$$\mathrm{Re} \int \frac{\varepsilon\ d\varepsilon\ e^{i\Phi} \exp[-2T(\varepsilon)/\tau_{Dwell}]}{\text{"phase-space"}}$$

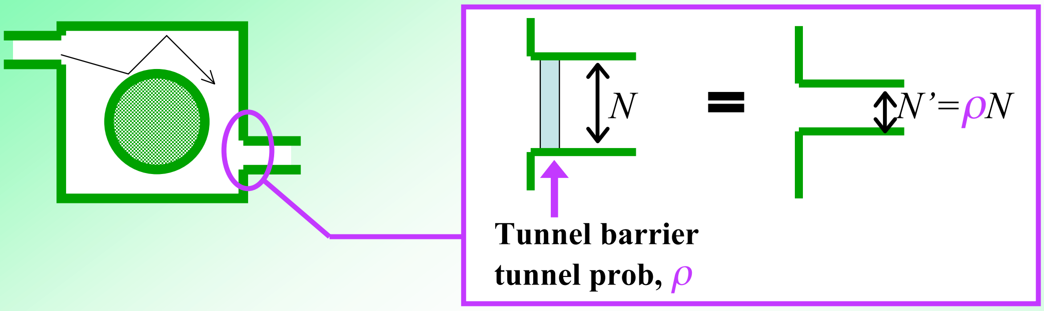
$T(\varepsilon) = \Lambda^{-1}\ln[1/\varepsilon^2]$

$T(\varepsilon \approx (\lambda_F/L)^{1/2})$ → **Ehrenfest time**

$$\text{Weak-loc} = \text{Drude conduct.} \times \left(- \frac{h\tau_{Dwell}}{\text{"phase-space"}} \exp[-\tau_E/\tau_{Dwell}] \right)$$

$$= - \frac{N_L N_R}{(N_L+N_R)^2} \exp[-\tau_E/\tau_{Dwell}]$$

TUNNEL-BARRIERS: weak-localization surprise



Drude conductance, $g_D \propto N'$ → ρN ✓

Weak–localization, $g_{wl} \propto [N']^0$ → ρ-independent ✗

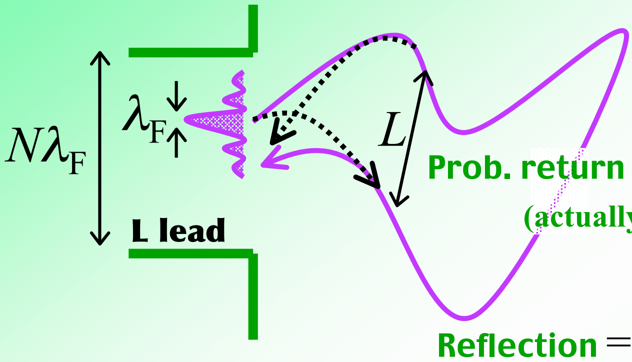
Random matrix theory
Iida *et al* (1990),
Brouwer-Beenakker (1996)
$g_{wl} \propto N^0 \times \rho$

Weak–localization ***vanishes*** for opaque barrier (for g_D = const.)

Tunnelling ***suppressing*** an interference effect

Coherent back-scattering peak

Jacquod-Whitney (2006)
Rahav-Brouwer (2006)



Prob. return approx ***doubled*** within λ_F
(actually in phase-space not real space)

Reflection $= N/2 \times [1 + 1/N - 1/(2N)]$

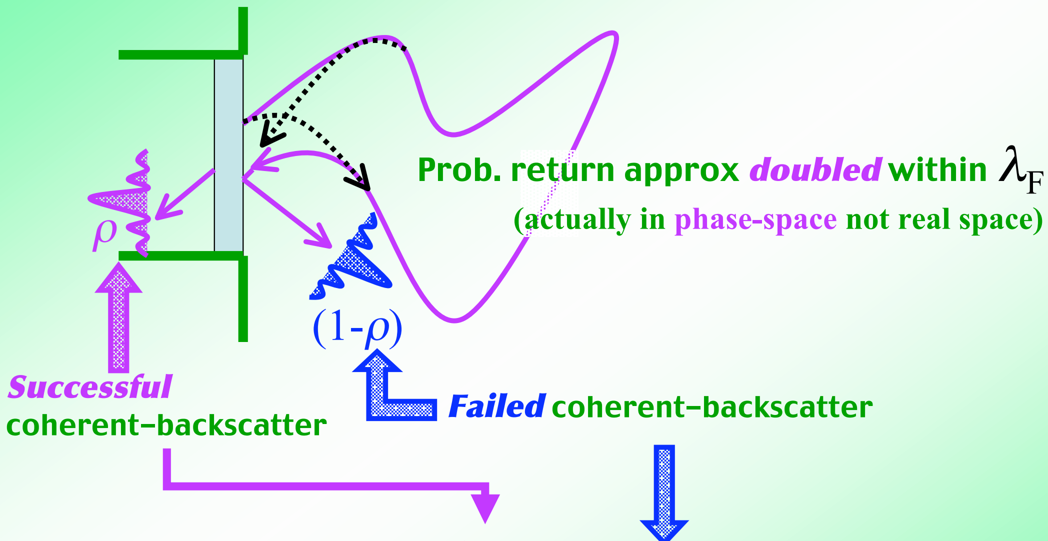
Weak–localization ≡ Reduced prob for all other paths,
half of which reflect

conductance = transmission $= N/2 \times [1 - 1/(2N)]$

Weak–localization $\sim N^0$

However **minimum time** for loop means
1 & 1 are actually $\exp[-\tau_E/\tau_D]$ $\xrightarrow{\text{classical limit}}$ 0

Coherent back-scattering with tunnel-barrier Whitney (2007)



Reflection $= N/2 \times [1 + (1/N)(\rho + (1-\rho)/2) - 1/(2N)]$

conductance $= N/2 \times [1 + (1/2N)(1-\rho) - 1/(2N)] \;= N/2 - 1/4 \times \rho$

✓ *Correct* weak-localization result $\sim N^0 \times \rho$

Weak-localization *suppressed* by tunnelling.

In both RMT regime and in cross-over to classical regime.

(even without tunnelling, weak-loc=0 *in* classical regime)

Suppression derived in RMT regime Iida *et al* (1990), Brouwer-Beenakker (1996)

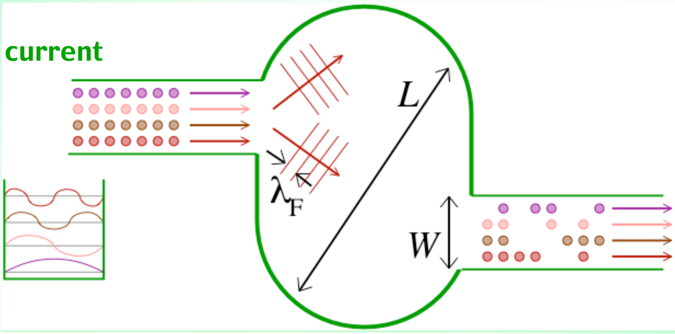
...but no physical picture of *why.*

Simple physical origin:

Tunnel-barriers *smear* coherent back-scattering peak
between reflection and transmission

Quantum Noise

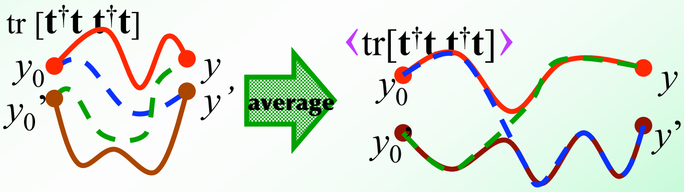
SHOT NOISE :
intrinsically *quantum* noise in current



Classically: *NO* noise
(determinism)

RMT to classical *paradox*

Fano factor: F = "noise/current" $= \dfrac{\langle \mathrm{tr}[\mathbf{t}^\dagger\mathbf{t} - \mathbf{t}^\dagger\mathbf{t}\,\mathbf{t}^\dagger\mathbf{t}]\rangle}{\langle \mathrm{tr}[\mathbf{t}^\dagger\mathbf{t}\,]\rangle}$

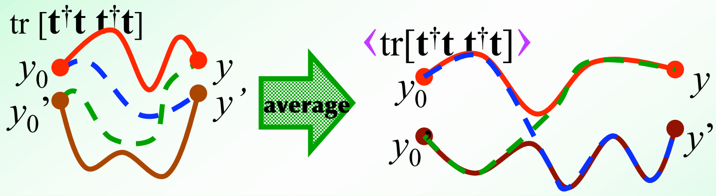


$F_{RMT} = \dfrac{N_L N_R}{(N_L+N_R)^2}$

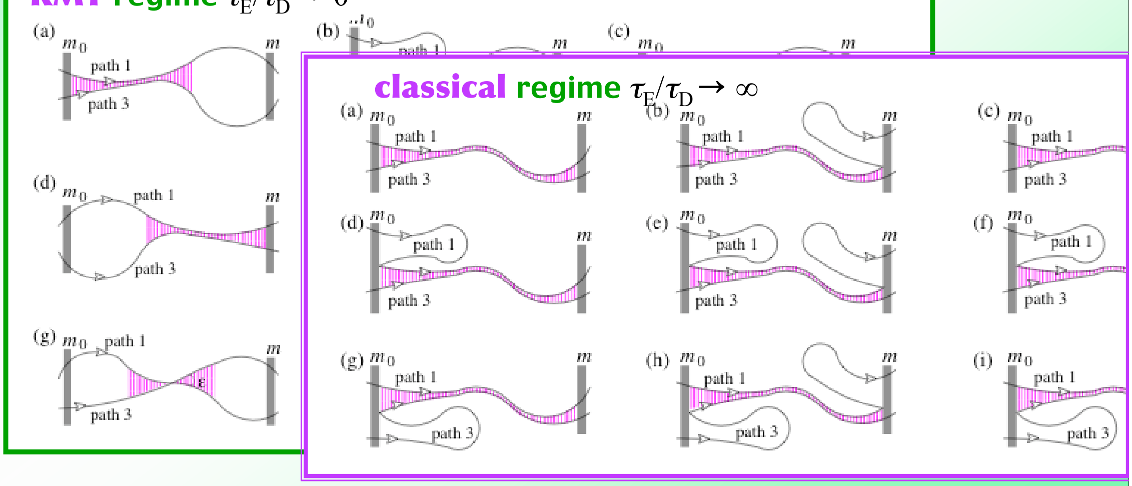
$N_L = N_R$ ➡ $F_{RMT} = 1/4$

SHOT NOISE : intrinsically *quantum* noise in the current (zero-temp, DC-current)

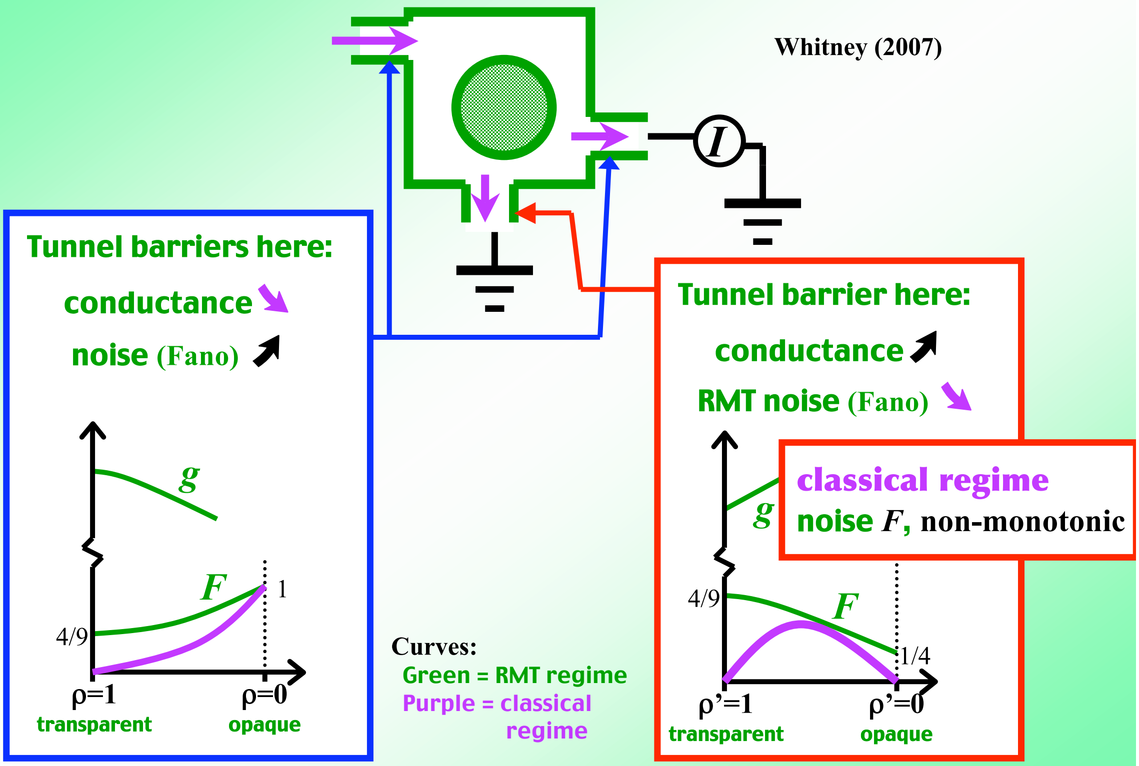
Fano factor: F = "noise/current" $= g^{-1}\langle \mathrm{tr}[\mathbf{t}^\dagger\mathbf{t} - \mathbf{t}^\dagger\mathbf{t}\,\mathbf{t}^\dagger\mathbf{t}]\rangle$



RMT regime $\tau_E/\tau_D \to 0$



Shot-noise for 3 leads with barriers



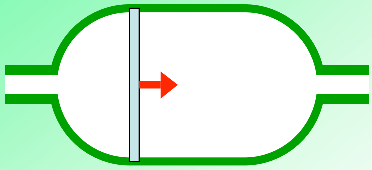
Shot noise conclusions

Interference and tunnel-barriers typically *enhance* noise
(less classical determinism)

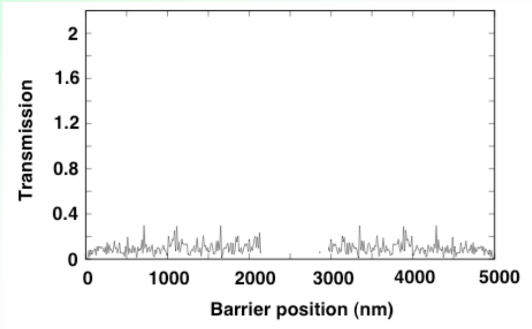
Noise (with tunnel barriers) in **classical** regime
is very different from **RMT** regime

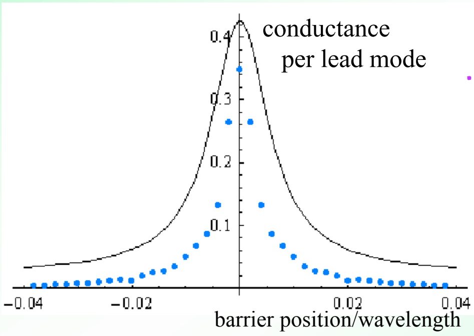
Barrier on 3rd lead → *enhances* (noise/current) in **RMT** regime
→ *reduce* or *enhance* (noise/current) in **classical** regime

Symmetry-induced conductance enhancement



conductance as function of barrier's *position*





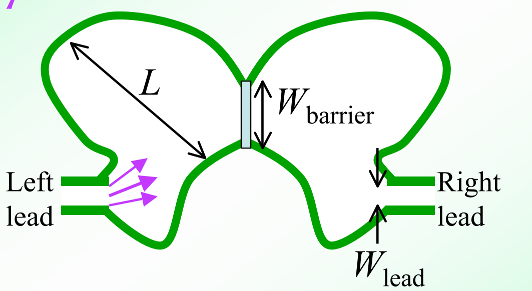
...so what is peak?

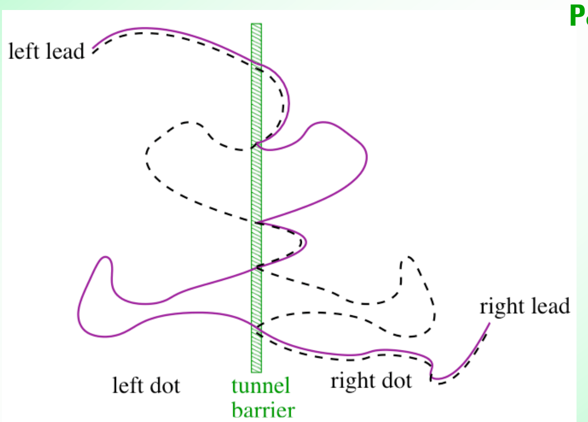
- ♣ mirror symm ➡ weak anti–loc?
  Baranger-Mello (1996) — *too small*
- ♣ resonant tunnelling?
  Bogomolny-Roubens (1998) — *no resonances*
- ♣ Fabry–Perot? — *no resonances*
- ♣ tunnelling into superconductor? *????*

Peak in butterfly double-dot?

"Butterfly" **double dot**
(Left↔Right mirror symmetry)

Lots **of chaos:** $W_{\text{lead}}, W_{\text{barrier}} << L$

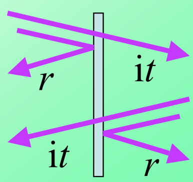




Paths in left and right dot have *same* properties
(amplitude, phase, etc)

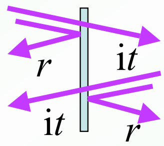
...except **scattering at barrier**

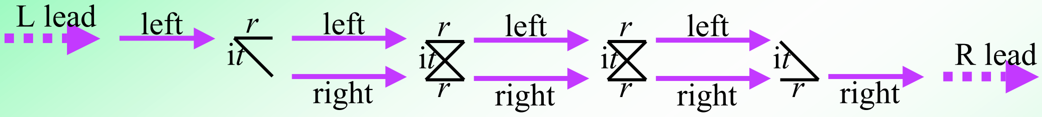
$$S_{\text{barrier}} = e^{i\varphi}\begin{pmatrix} r & it \\ it & r \end{pmatrix}$$



Summing all contributions

$$S_{\text{barrier}} = e^{i\varphi}\begin{pmatrix} r & it \\ it & r \end{pmatrix}$$



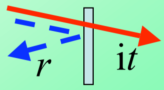


...but conductance is a double sum over paths

so we double scattering space

$$(S_{\text{barrier}} \otimes S^{\dagger}_{\text{barrier}}) \Rightarrow \tilde{\mathbb{S}} = \begin{pmatrix} r^2 & -iB_a rt & iB_a^* rt & t^2 \\ -iB_a rt & B_a^2 r^2 & |B_a|^2 t^2 & iB_a rt \\ iB_a^* rt & |B_a|^2 t^2 & B_a^2 r^2 & -iB_a^* rt \\ t^2 & iB_a rt & -iB_a^* rt & r^2 \end{pmatrix}$$

Scattering from left:left to right:left



$B_a \equiv$ ***all* quantum asymmetries**

classically small L-R asymm, weak magnetic field, decoherence, etc

Geometric series in number of "barrier hits"

Left dot ≡ right dot ⇒ Amplitudes: $A_{\text{left}} A^*_{\text{right}} = |A_{\text{left}}|^2$ ⇒ probability

Phases *cancel*: $(S_{\text{left}} - S_{\text{right}}) = 0$

4x4 matrix $\tilde{\mathbb{S}}$

$$g = N \times (1-P)\sum_{n=0}^{\infty} P^n \left[\tilde{\mathbb{S}}^n\right]_{41} = N \times \sum_{j=1}^{4} \mathbb{U}_{4j} \frac{1-P}{1-P\Lambda_j}[\mathbb{U}^{-1}]_{j1}$$

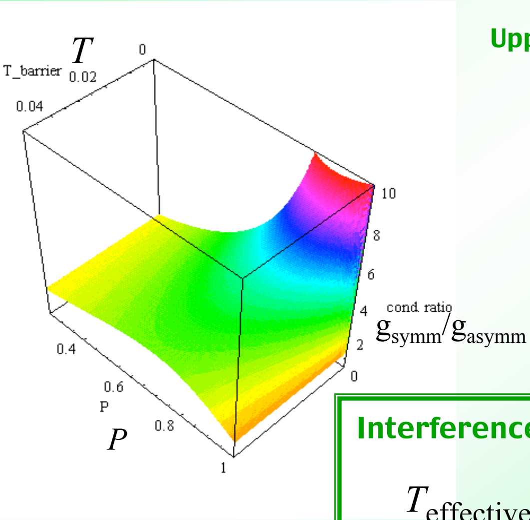
- $(1-P)$: prob. to hit lead
- $\sum_{n=0}^{\infty}$: sum over number of barrier hits, n.
- P^n: prob. to hit barrier
- $\sum_{j=1}^{4}$: sum over 4 eigenvalues
- $\mathbb{U}$: Eigenvectors of $\tilde{\mathbb{S}}$
- Λ_j: Eigenvalues of $\tilde{\mathbb{S}}$

$$B_a = \begin{cases} 0 & \textit{no}\ \text{symmetry} \Rightarrow \text{classical conductance} \rightarrow \tilde{\mathbb{S}} = \begin{pmatrix} r^2 & 0 & 0 & t^2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ t^2 & 0 & 0 & r^2 \end{pmatrix} \\ 1 & \textit{perfect}\ \text{symmetry \& no decoherence} \rightarrow \tilde{\mathbb{S}} = (S_{\text{barrier}} \otimes S^{\dagger}_{\text{barrier}}) \end{cases}$$

Height of conductance peak?

Tunnelling rate, $T \to 0$

& then $W_{\mathrm{lead}}/W_{\mathrm{barrier}} \approx (1\text{-}P) \to 0$ ➡ $g_{\mathrm{symm}}/g_{\mathrm{asymm}} \to \infty$



Upper–bound is conductance without barrier:

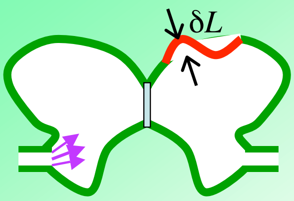
$g_{\mathrm{symm}} \leq N/2$

In principle : ***NO*** **upper–bound on** N

Interference *suppresses* tunnel–barriers

$$T_{\mathrm{effective}} = \frac{(1+P)T}{1\text{-}P + 2PT} \quad \to 1 \text{ for } P \to 1$$

Lorentizian peak shape



Average phase difference = 0

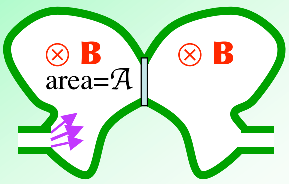
Real $B_{\mathrm{a}} = (1\text{-}\Gamma)^{-1/2}$ **can diagonalize** $\tilde{\mathbf{S}}$

Γ= var[phase difference]

$\propto \tau_D/\tau_\varphi$ **for decoherence**

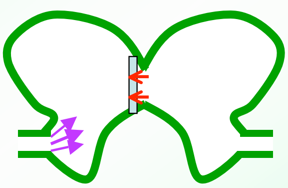
$\propto (\delta L/\lambda_F)^2\,(\tau_D/\tau_0)$ **for deformation (top left)**

$\propto (\phi_B \mathcal{A})^2\,(\tau_D/\tau_0)$ **for B–field (middle left)**



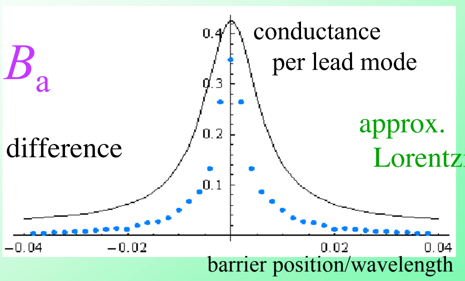
$$g = g_{\mathrm{asymm}} + \frac{g_{\mathrm{symm}}\text{-}g_{\mathrm{asymm}}}{1\text{-K}}$$

$$K = \frac{P(1-2T_{\mathrm{b}})\Gamma\tau_{\mathrm{D}}}{1+P^2-2P(1-2T_{\mathrm{b}})}$$

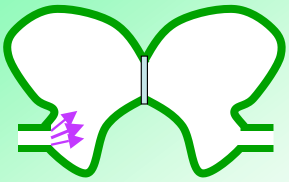


Complex B_{a}

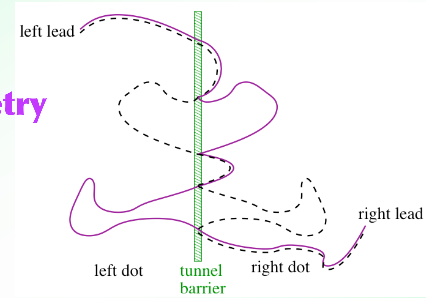
Non-zero
average phase difference



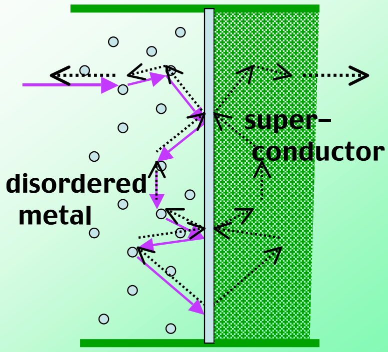
Similarities:- reflectionless tunnelling into superconductor



left-right symmetry



electron-hole symmetry
(Andreev reflection)



No semiclassical theory ...but *easy to guess!*

e → left dot
-h → right dot

Similar to above but *not* the same.

Unaffected by distortion, B-field (diffuson-like)
Only works at Fermi surface – limited to *small* voltage-bias

Conclusions

Tunnel-barriers *suppress* weak-localization (interference)

Smearing of coherent back-scattering peak
between reflection and transmission

Interference and tunnel-barriers → quantum noise

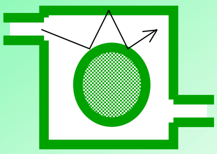
Barrier on 3rd lead → *enhances* (noise/current) in RMT regime
→ *reduce* or *enhance* (noise/current) in classical regime

Mirror-symmetry: Interference *suppresses* tunnel-barrier

Huge enhancement of conductance for nearly *opaque* barriers

Blank

Semiclassics : *ray optics* for the 21st century



scattering matrix:

$$S = \begin{pmatrix} r & t^\dagger \\ t & r' \end{pmatrix}$$

Baranger *et al* (1993)

Van Vleck/Gutzwiller propagator = *ray optics*

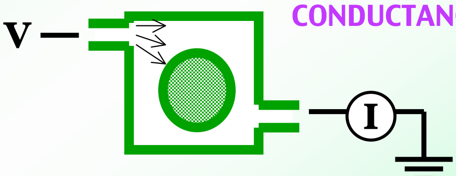
Sum over all classical paths: S_γ = classical action

$$t_{nm} \sim \mathcal{A} \sum_\gamma A_\gamma \exp[i\, S_\gamma / h]$$

Coupling to lead mode wavefunctions

$$\mathcal{A} \sim \int dy_0 \int dy \; \langle n|y\rangle \langle y_0|m\rangle$$

Landauer–Buttiker



CONDUCTANCE : $G = I/V = (e^2/h)\, \mathrm{tr}[\mathbf{t}^\dagger \mathbf{t}]$

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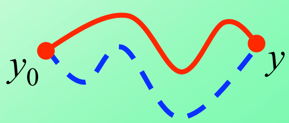

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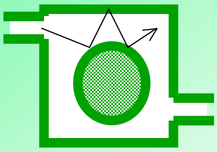

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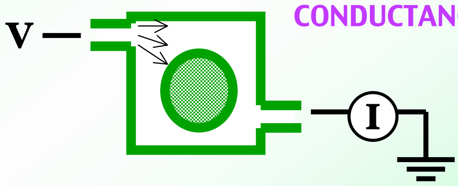

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