Institut Laue Langevin, Grenoble, France.

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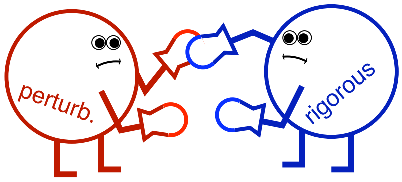
Which theory for dissipation in quantum systems (such as qubits) ?

Robert S Whitney



Discussions:

J. Siewert (Regensburg)
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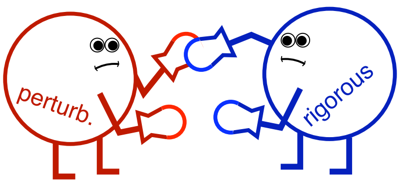
A. Shnirman (Karlsruhe)
M. Clusel (NYU)
M. Hall (Australian Nat. Univ.)

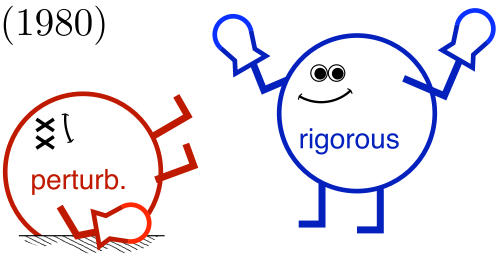
Reference: R.W., J. Phys. A: Math. Theor. **41**, 175304 (2008)

Overview — Two theories of dissipation

[1] *Phenomenological* method Lindblad (1976) — rigorous
[2] *Microscopic* method Bloch-Redfield (1957) — perturbative

RESULTS DISAGREE Dumcke-Spohn (1980)



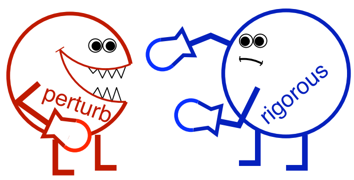


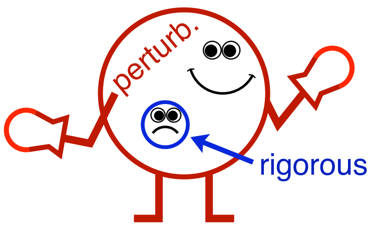
YET Bloch-Redfield (perturb.) remains *most popular* theory for solid-state qubits!

Minor improvement to perturb. method

More powerful

Less user-friendly



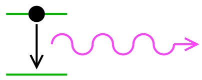


Why dissipative quantum mechanics?

No quantum system is isolated $\Leftrightarrow$ energy exchange

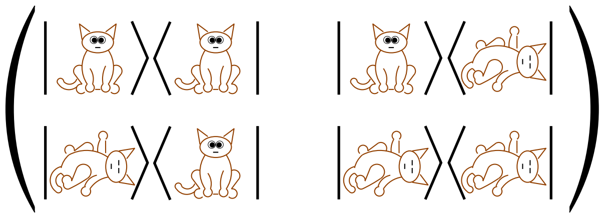
Dissipation: common in *quantum* world as in classical world

- Quantum optics: decay of excited atomic state



- Chemical physics: most reactions $2Na_2(s)+2HCl(aq)\rightarrow 2NaCl(aq)+H_2(g)$
- Statistical physics: what is equilibrium?
- Solid-state: Resistance in nanoscale circuits
- Quantum information: Decoherence of qubits
- Philosophy: No Schrödinger cats in everyday life

density matrix $\neq$

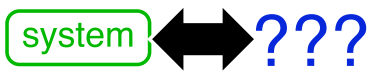


Summary of previous works

Phenomenological method

- Know nothing about environ.
- Know system dynamics are physical

$\Rightarrow$ Probabilities are real, positive and sum to one

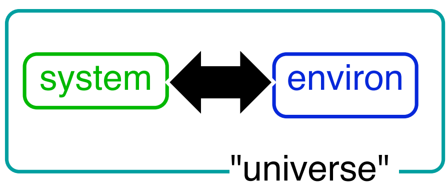


$+$ rigorous

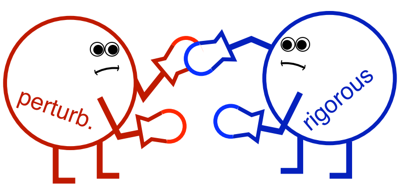
Microscopic method

- Know everything about environ.

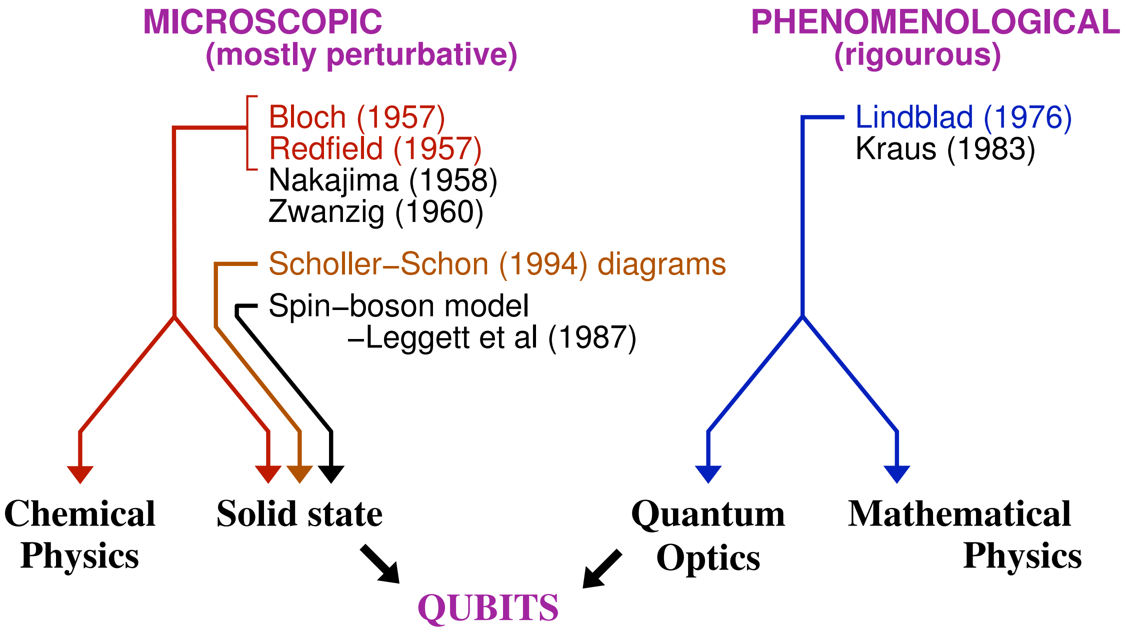
$\Rightarrow$ know $\hat{\mathcal{H}}_{\mathrm{universe}}$



$+$ typically perturbative

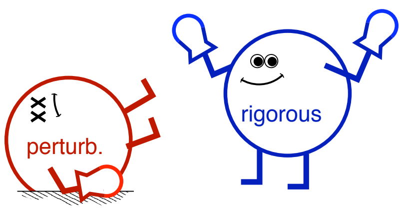


Road-map of previous works



+ EXACTLY SOLUBLE MODELS (non-generic)

Why use the perturbative method?



...but rigorous method is phenomenological

cf. superconductor: Landau-Ginzberg vs. BCS

Only a microscopic theory can answer certain questions:

- dependence on environ. temperature?
- dependence on environ. spectrum?
- How do we engineer system to *minimize* decoherence?

Perturbative method *usually* gives "plausible" results
but it sometimes generates *negative* probabilities
...so can we really trust it??

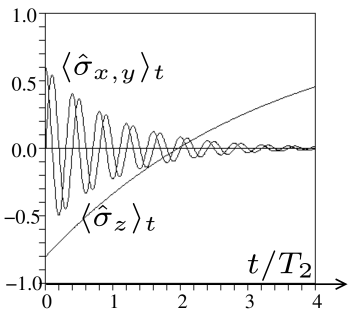
Density-matrix and Bloch-sphere

Observable: $\langle\hat{O}\rangle_t = \mathrm{tr}\big[\hat{O}\,\hat{\rho}(t)\big]$

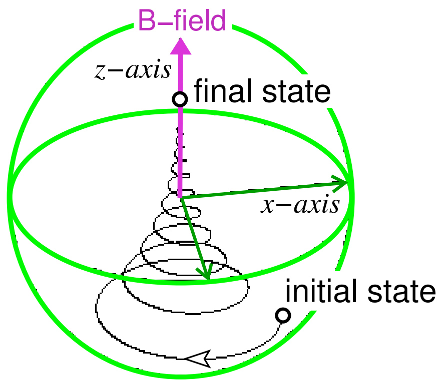
Evolution: $\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\mathrm{i}\Big[\hat{\mathcal{H}},\hat{\rho}(t)\Big] +$ dissipation

Any two-level system $\equiv$ spin-half

$$\hat{\rho}(t) = \frac{1}{2}\begin{pmatrix} 1+\langle\hat{\sigma}_z\rangle_t & \langle\hat{\sigma}_x\rangle_t - \mathrm{i}\langle\hat{\sigma}_y\rangle_t \\ \langle\hat{\sigma}_x\rangle_t + \mathrm{i}\langle\hat{\sigma}_y\rangle_t & 1-\langle\hat{\sigma}_z\rangle_t \end{pmatrix}$$



$\Rightarrow$ plot $\langle\hat{\sigma}_x\rangle_t$ on x-axis
... etc



Lindblad's master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\mathrm{i}\big[\hat{\mathcal{H}},\hat{\rho}(t)\big] - \mathcal{L}[\hat{\rho}(t)]$$

For set of "orthogonal" and "normalized" operators, $\hat{L}_n$s.,

$$\mathcal{L}[\hat{\rho}(t)] \equiv \sum_n \lambda_n \Big(\hat{L}_n^\dagger\hat{L}_n\hat{\rho}(t) + \hat{\rho}(t)\hat{L}_n^\dagger\hat{L}_n - 2\hat{L}_n\hat{\rho}(t)\hat{L}_n^\dagger\Big)$$

with ***no*** negative λ_ns

Lindblad proved: ***All*** other master equation are ***unphysical***
— negative probabilities

Rigorous proof based on following ***postulates***:

- Evolution continuous in time
- Eqn. translationally invarient in time
- $\cdots$
- ***physical*** $\equiv$ "complete positivity"

Understanding Lindblad eqn.

Eqn. is remarkable simple!!

"Markovian" — evolution is function of $\hat{\rho}(t)$ not $\int \mathrm{d}t' \hat{\rho}(t')(\ldots)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\mathrm{i}\left[\hat{\mathcal{H}}, \hat{\rho}(t)\right] - \mathcal{L}[\hat{\rho}(t)]$$

Example: spin-half with one env.-coupling $\hat{L}_1 = \hat{\sigma}_z$

$$\mathcal{L}[\hat{\rho}(t)] \equiv \lambda\Big(2\hat{\rho}(t) - 2\hat{\sigma}_z\hat{\rho}(t)\hat{\sigma}_z\Big) = 2\lambda \begin{pmatrix} 0 & \langle\hat{\sigma}_x\rangle - \mathrm{i}\langle\hat{\sigma}_y\rangle \\ \langle\hat{\sigma}_x\rangle + \mathrm{i}\langle\hat{\sigma}_y\rangle & 0 \end{pmatrix}$$

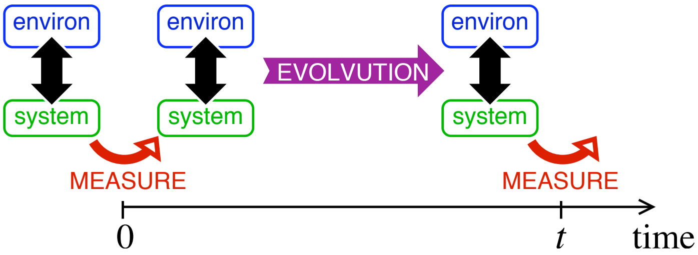
In General:

For $\lambda_n > 0$ — decay in all directions $\perp$ to $\hat{L}_n$

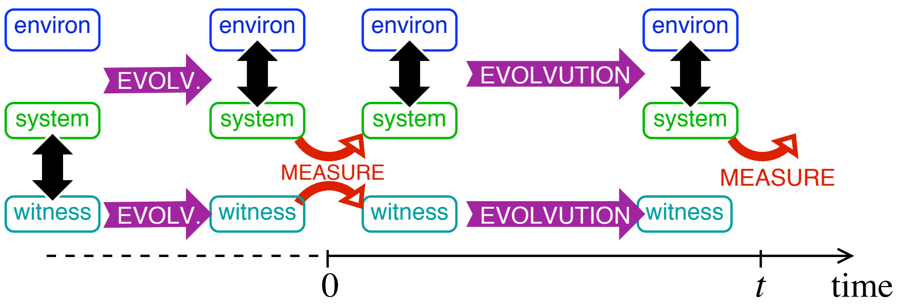
... but for $\lambda_n < 0$ — growth

Positivity and complete positivity

Positivity



Complete positivity



All *completely positive* dynamics are also *positive*

Ocassionally the two are equivalent – as for my 2-level system model

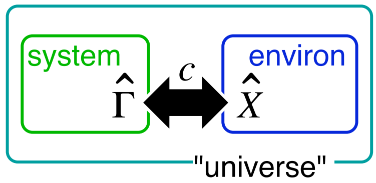
M. Hall, arXiv:0802.0606

Bloch-Redfield's master equation I

Hamiltonian:

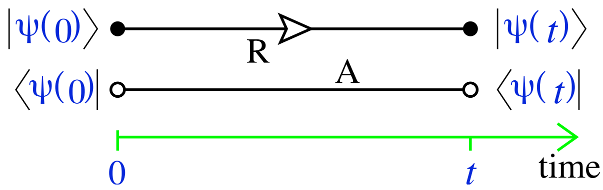
$$\hat{\mathcal{H}}_{\rm univ} = \hat{\mathcal{H}}_{\rm sys} + c\hat{\Gamma}\hat{X} + \hat{\mathcal{H}}_{\rm env}$$

perturbation



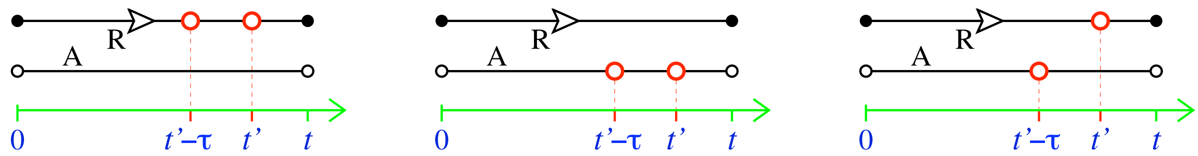
Evolution: $\hat{\rho}(t) = \exp[-\mathrm{i}\hat{\mathcal{H}}_{\rm univ}t]\ \hat{\rho}(0)\ \exp[\mathrm{i}\hat{\mathcal{H}}_{\rm univ}t]$

$$\hat{\rho}(t) = |\psi(t)\rangle\langle\psi(t)|$$

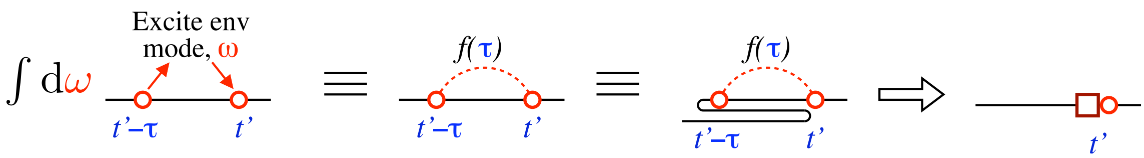


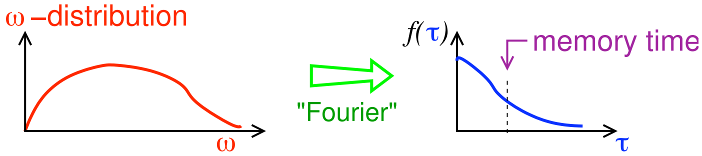
Second-order perturbation theory:

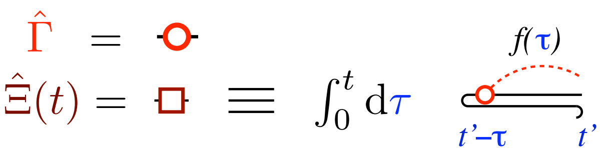
2nd order in $\exp[-\mathrm{i}\hat{\mathcal{H}}_{\rm univ}t]$ — 2nd order in $\exp[\mathrm{i}\hat{\mathcal{H}}_{\rm univ}t]$ — 1st order in both



Bloch-Redfield's master equation II







Dissipative part of $\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t)$

$$\mathcal{L}[\hat{\rho}(t)] = c^2\Big(\hat{\Gamma}\hat{\Xi}(t)\hat{\rho}(t) + \hat{\rho}(t)\hat{\Xi}^\dagger(t)\hat{\Gamma}^\dagger - \hat{\Xi}(t)\hat{\rho}(t)\hat{\Gamma}^\dagger - \hat{\Gamma}\hat{\rho}(t)\hat{\Xi}^\dagger(t)\Big)$$

Bloch-Redfield in Lindblad's form

[spin-1/2 – Dumcke-Spohn (1979)]

Dissipative part of $\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t)$

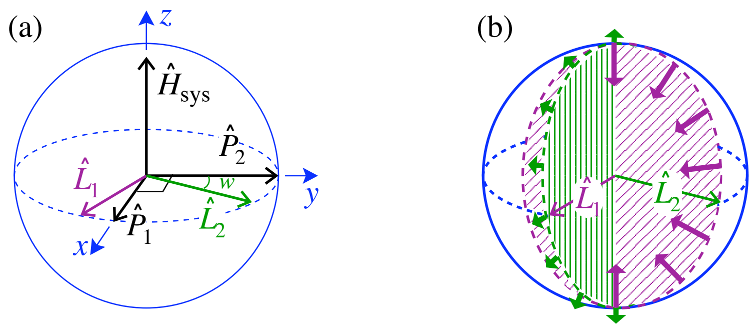
$$\mathcal{L}[\hat{\rho}(t)] = c^2\Big(\hat{\Gamma}\hat{\Xi}(t)\hat{\rho}(t) + \hat{\rho}(t)\hat{\Xi}^\dagger(t)\hat{\Gamma}^\dagger - \hat{\Xi}(t)\hat{\rho}(t)\hat{\Gamma}^\dagger - \hat{\Gamma}\hat{\rho}(t)\hat{\Xi}^\dagger(t)\Big)$$

- Operators not orthogonal (unlike Lindblad) $\Rightarrow$ ORTHOGONALIZE
- Cross coupling $\Rightarrow$ DIAGONALIZE

$$\mathcal{L}[\hat{\rho}(t)] = \sum_{n=1,2} \lambda_n \Big(\hat{L}_n^\dagger \hat{L}_n \hat{\rho}(t) + \hat{\rho}(t)\hat{L}_n^\dagger \hat{L}_n - 2\hat{L}_n \hat{\rho}(t)\hat{L}_n^\dagger\Big)$$

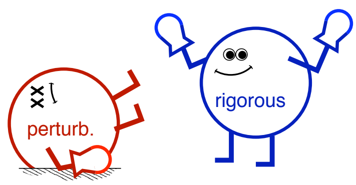
...same as rigorous eqn. with $\lambda_2 \propto -c^2$ $\Leftarrow$ ***always*** negative

2-level system
with $\hat{\mathcal{H}} \propto \hat{\sigma}_z$
& $\hat{\Gamma} = \hat{\sigma}_x$



Obituary for perturbative method 1957-1979

Rigorous theory says
"negative λ means negative probs."



We have $\lambda_2 < 0$ for any c
$\Rightarrow$ perturbative method unphysical

Numerics confirm negative probabilities - Gaspard & co-workers

Resurrection of perturbative method

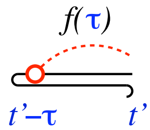
Forgot $\hat{\Xi}(t)$ is time-dependent

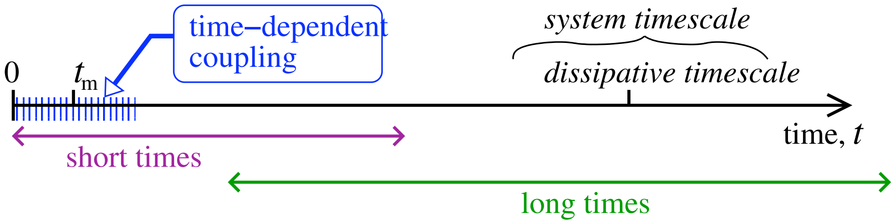
- Invalidates rigorous proof !! *Assumption* that eqn. is time-indep.
- Numerics change with t-depend. – Gaspard & co-workers

OPEN QUESTION: Does perturb. method ***avoid*** negative probs. ??

Time-dependence of parameters

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\mathrm{i}\left[\hat{\mathcal{H}}, \hat{\rho}(t)\right] - \mathcal{L}[\hat{\rho}(t), t]$$

Time-dependent $\mathcal{L}[\hat{\rho}(t), t]$ since $\hat{\Xi}(t) = \square \equiv \int_0^t \mathrm{d}\tau$ 



Analogy: without matrix structure

$$\frac{\mathrm{d}}{\mathrm{d}t}y(t) = (\mathrm{i}h - F(t))y(t)$$

where $F(t) \to f$ for $t \gg$ memory time

Approx: $\frac{\mathrm{d}}{\mathrm{d}t}y(t) = (\mathrm{i}h - f)y(t)$

Trivial to solve, but incorrect for $t \sim$ memory time

Proving positivity for short memory-times

... continue analogy $\quad \frac{\mathrm{d}}{\mathrm{d}t}y(t) = (\mathrm{i}h - G(t))y(t)$

[I] Short-times $t \ll 1/G(t)$

$$y(t) \simeq \left[\mathrm{e}^{iht} - \int_0^t \mathrm{d}t' \mathrm{e}^{ih(t-t')} G(t') \mathrm{e}^{-iht} + \mathcal{O}[G^2]\right] y(0)$$

[II] Long-times $t > t_0 \gg$ memory-time

$$\frac{\mathrm{d}}{\mathrm{d}t}y(t) \simeq (ih - g)y(t) \qquad \Rightarrow y(t) \simeq \mathrm{e}^{(ih-g)(t-t_0)} y(t_0) + \mathcal{O}[G-g]$$

Large overlap of regimes if memory time $<< 1/|G(t)| \sim c^{-2}$

Do same with ***matrix*** eqn. for $\hat{\rho}(t)$

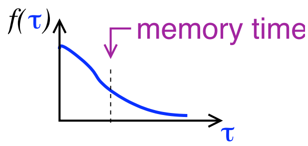
Check purity is not greater than ***ONE*** $\Rightarrow$ No negative probs.

i.e. find maxima of purity

— constraint: initial state is physical

...but NEED form for $f(\tau)$ cf. $G(t)$ above

Memory funct.



Two-level system

Simplest system: Two-level system

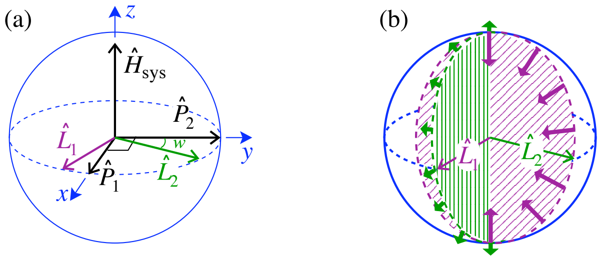
Simplest environment: [a] smooth very-broad spectrum of excitations

[b] high temperature

$\Rightarrow$ Memory time $\ll$ system dynamics

Simplest coupling: $\hat{\Gamma} = \hat{\sigma}_x$ $\quad \Rightarrow \hat{\mathcal{H}}_{\rm univ} = -B\hat{\sigma}_z + c\hat{\sigma}_x\hat{X} + \hat{\mathcal{H}}_{\rm env}$

Proven : NO negative probabilities



Example: initial state $= |\uparrow_x\rangle$

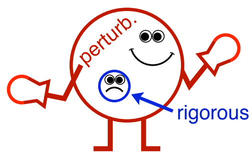
neglect t-depend.

purity$= 1 + c^2 t$

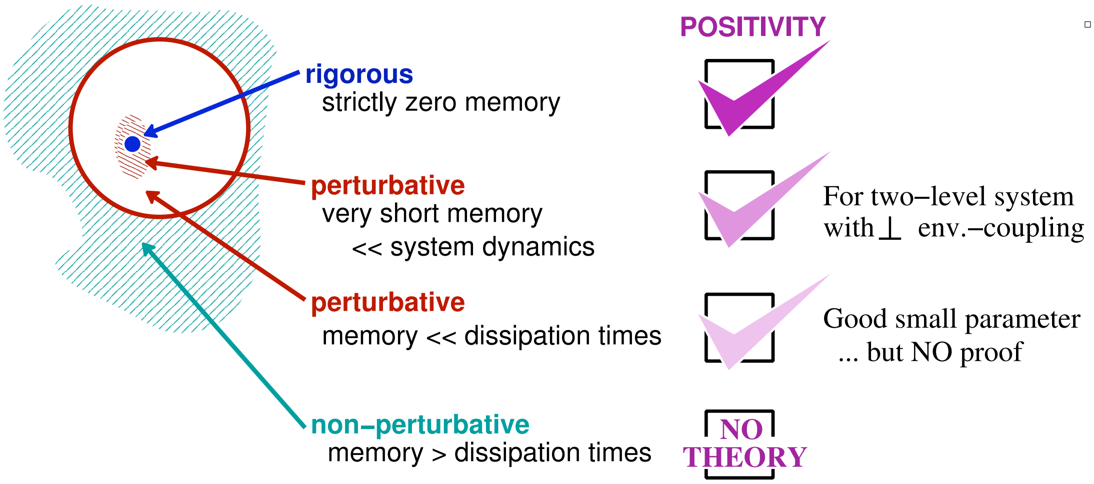
keep t-depend.

purity$= 1 - c^2 t^3/t_{\rm m}^2$

Conclusions



Regimes of applicability of theories:



... but NEED to keep ***time-dependence*** in perturbative method

+ perturbative method is microscopic

Enabling study of how dissipation is affected by environment details (temperature, spectrum, etc)