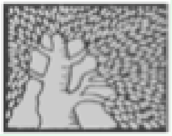

Geometric phases, adiabaticity and decoherence in *qubits*

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Mini-atelier cohérence et information quantiques. Grenoble, Juin 2006

Who cares about qubits?

Fully controllable quantum systems:

A PLAY-GROUND FOR QUANTUM PHYSICS

STUDY interplay of **quantum** effects with **decoherence**
when we have **good control of BOTH**.

Quantum effects: ♣ super-positions ♣ interference
♣ entanglement ♣ tunnelling

Control:
♦ system's **Hamiltonian** (including inter-qubit couplings)
♦ excellent **read-out** of system state.

♣ environment coupling strength and *operator*
♣ environment *spectrum*

IF we have thousands of qubits, we **MIGHT** have a **quantum computer**

Overview

- la phase de Berry POUR LES NULS : abelian and non-abelian
- Berry phase in qubits

DECOHERENCE: when system is **not isolated**

- phase information in adiabatic limit for non-isolated system?

  Naïve arguments say NO
  Experiments say YES
  } Resolving contradiction

- Berry phase modified by environment :

  remains geometric (not monopole) also complex ⇒ *geometric dephasing*

- geometric phases to avoid decoherence? *No!*
- symmetries to avoid decoherence? decoh-free subspace, Saclay "quantronium"

- geometric/topological/holonomic quantum computing :

  No such thing as a free lunch !!

What is Berry phase ?

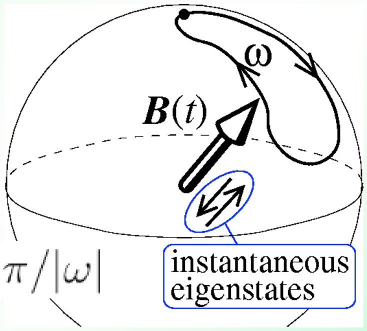
Berry (1984)

Slowly varying B–field :

rate of rotation, $|\omega| << |\boldsymbol{B}|$

$P(|\uparrow\rangle \rightarrow |\downarrow\rangle\,) \sim \omega/B \;\Rightarrow\; 0$

i.e. Adiabatic evolution



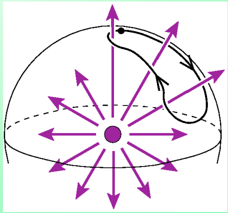
$t \sim 2\pi/|\omega|$

$$\Phi = \Phi_{\rm Dynamic} + \Phi_{\rm Berry} + \mathcal{O}[(Bt)^{-1}]$$

$$\Phi_{\rm Dynamic} = \tfrac{1}{2}|\boldsymbol{B}|t$$

$$\Phi_{\rm Berry} = \tfrac{1}{2}\text{(enclosed solid angle)}$$

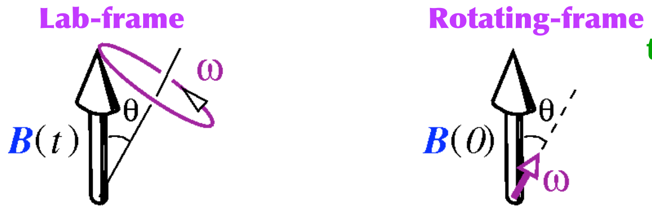
$$= \tfrac{1}{2}\text{(flux of monopole thru loop)}$$



La phase de Berry pour les nuls

Berry (1987)

⇒ **Rotating frame** — **rotates with B-field :** **Hamiltonian** ≈ **time-indep.**



time-indep. Hamiltonian

total field = $B+\omega$

energy gap = $|B+\omega|$

$t = 2\pi n/|\omega|$

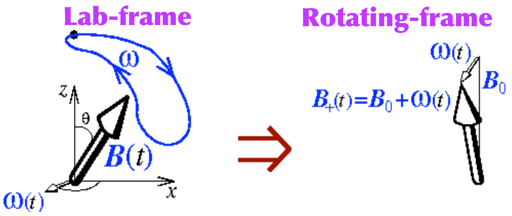
Total phase = $|B+\omega|\,t = |B|t + |\omega|\,t\cos\theta$

dyn. phase

Berry phase

In general:

$$\mathcal{H}_{\rm rot} = i[\partial U/\partial t]\,U^{-1} + U\,\mathcal{H}_{\rm lab}\,U^{-1}$$



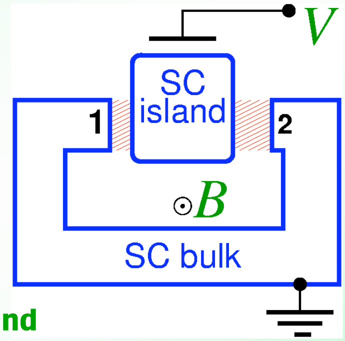
Berry phase is SHIFT of levels due to pseudo-forces/fields

Berry phase in qubit systems?

Potential solid-state realisation

Berry phase in Superconducting Nanocircuit (qubit) Falci *et al* (2000)

$$\mathcal{H} = E_C(n-n_V)^2 - E_J(B)\cos[\theta-\alpha_B]$$



Consider only lowest 2 charge-states of island

$$|{\uparrow}\rangle \equiv |n\rangle \quad \& \quad |{\downarrow}\rangle \equiv |n+1\rangle$$

Reduced Hamiltonian: $\hat{\mathcal{H}} = \begin{pmatrix} E_J\cos(\alpha_B) \\ E_J\sin(\alpha_B) \\ E_C(1-n_V) \end{pmatrix} \cdot \hat{\sigma}$

Environment?
- ♣ **charge fluctuations couple via** σ_z
- ♣ **current fluctuations couple via** σ_x, σ_y

Non-abelian geometric phase (Berry phase of degenerate levels)

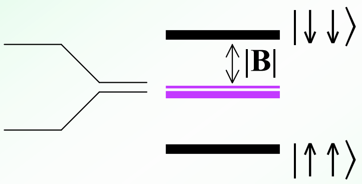
Wilczek-Zee (1984)

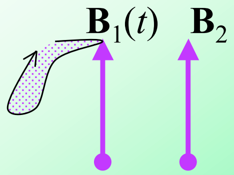
... POUR LES NULS

Two spin−1/2 in same field ➠ two degenerate levels

$$|\psi_0\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

$$|\psi_1\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$





Now do “Berry” to one spin (with $|\mathbf{B}_1(t)| = |\mathbf{B}_2|$)

dynamic phases cancel ➠ total phase = Berry phase

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \xrightarrow{\text{adiab}} e^{i\Phi_{\text{Berry}}}|\uparrow\downarrow\rangle - e^{-i\Phi_{\text{Berry}}}|\downarrow\uparrow\rangle$$

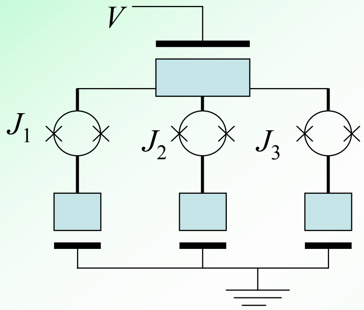
$$= \cos\Phi_{\text{Berry}}\,|\psi_0\rangle + i\,\sin\Phi_{\text{Berry}}\,|\psi_1\rangle$$

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \xrightarrow{\text{non-adiab}} a_1|\uparrow\downarrow\rangle - a_2|\downarrow\uparrow\rangle + (\omega/\mathrm{B})[a_3|\downarrow\downarrow\rangle - a_4|\uparrow\uparrow\rangle]$$

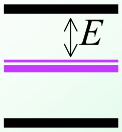
Non-abelian Berry phase in SC qubits

Faoro-Siewert-Fazio (2003)

Potential experiment:



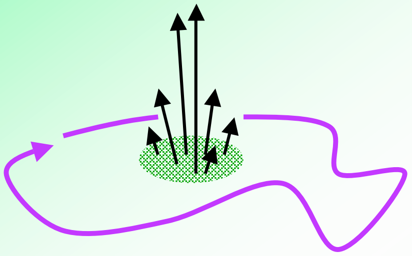
Energy levels



Topological Berry phase

For complicated Hamiltonians : Berry flux through path not uniform

i.e. in rotating frame : "ω" is complicated function of Hamiltonian's parameters



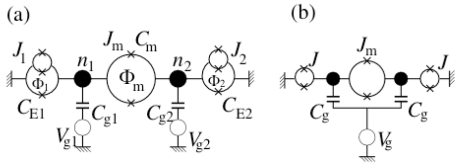
Localised Berry flux

⇨ Topological Berry phase

ONLY winding number matters

Potential expt:
Cholascinski 2005

Two SC charge qubits



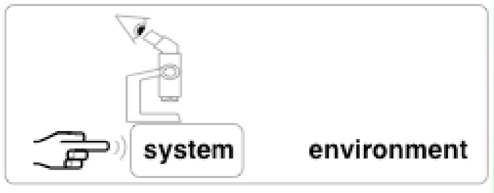
Can also use effect to make **quantised** charge pump

DECOHERENCE:
quantum physics for non-isolated systems

Define :

"SYSTEM" = degrees of freedom that we **control/measure**
"ENVIRONMENT" = all other degrees of freedom

Universe =



System's reduced density matrix : $\rho_{\text{system}} = \text{Tr}_{\text{env}}[\rho_{\text{universe}}]$

Universe : Hamiltonian evolution
System : **dissipative** evolution

Models of system + environment

❶ Spin coupled to quantum environment

$$\mathcal{H}_{\text{univ}} = \mathcal{H}_{\text{syst}} + \mathcal{H}_{\text{interaction}} + \mathcal{H}_{\text{env}}$$

$\mathcal{H}_{\text{syst}} = -\frac{1}{2}\boldsymbol{B}(t).\boldsymbol{\sigma}$ $\mathcal{H}_{\text{interaction}} = -\sum_n C_n(a_n^\dagger + a_n)\,\sigma_z$

where $a_n^\dagger, a_n$ create/annihilate nth environment mode

Example : **spin-boson model** Leggett *et al* (1986,87)

Environment = oscillators with smooth spectral distrib.

❷ Spin coupled to classical coloured noise

$\mathcal{H}_{\text{syst}} = -\frac{1}{2}(\boldsymbol{B}(t)+\boldsymbol{K}(t)).\boldsymbol{\sigma}$ where $\langle \boldsymbol{K}(t)\ \boldsymbol{K}(0)\rangle = C^2 f(t)$

Equiv. to quantum for many quantities: T_1, T_2, Lamb shift, etc

Caldeira-Leggett(1983), Whitney-Makhlin-Shnirman-Gefen (2004)

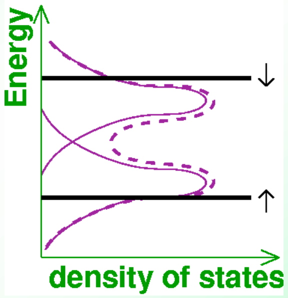
Berry phase with dephasing?

Environment induces level-broadening

⇒ No Gap

$$P(|\uparrow\rangle \rightarrow |\downarrow\rangle) \rightarrow 1 \quad \text{as} \quad t \rightarrow \infty$$

No Adiabaticity ⇒ No Berry phase



Whitney–Gefen, *PRL* (2003)

BUT : All real expts are non-isolated, yet Berry phase is observed

Berry phase is observable whenever

adiabatic time << dephasing time

$$\hbar/E_{\text{gap}} \ll T_2$$

i.e. require small matrix elements for transitions not a true gap

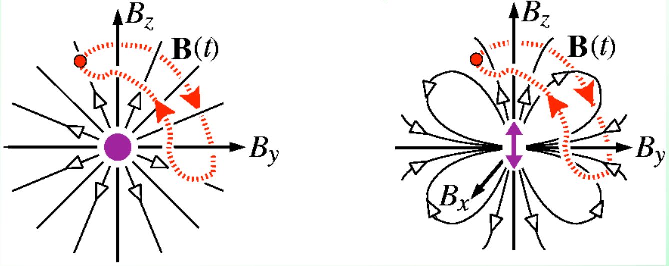
Env.-induced modification of the Berry phase

get phase as $\oint$ along path of $B(t)$ ⇒ use Stokes' theorum

⇒ surface int. $\Phi_{\text{Berry}} = \int \mathrm{d}\mathbf{S}\cdot(\mathbf{b}+\delta\mathbf{b})$

monopole
pseudo-field b

"quadrupole"
pseudo-field δb



Angular = $Y_{20}(\theta,\varphi)$
Radial ≠ B^{-4}

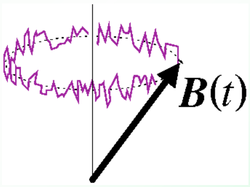
(non-zero curl)

Amplitude of monopole = 1/2

Amplitude of "quadrupole" = $C^2 \times$ complex function (env. spectrum)

How do we get these results?

♣ Toy problem : Noisy classical field

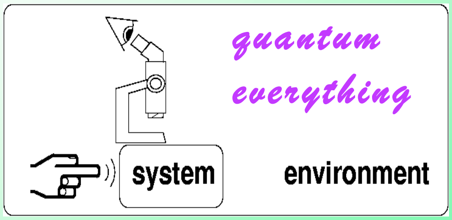


Whitney–Gefen, *Proc. Moriond* (2001)
Whitney–Makhlin–Shnirman–Gefen, *Proc. NATO–ARW* (2004)

♣ Fully quantum problem :

coupling many environ. modes, trace them out

Use rotating frame trick



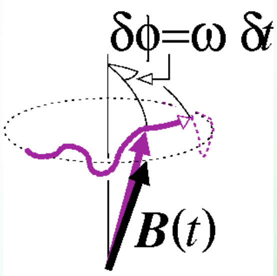
Whitney–Gefen, *PRL* (2003)
Whitney–Makhlin–Shnirman–Gefen,
PRL (2005)

Noisy classical field

Toy problem : Gaussian white-noise Whitney-Gefen (2001)

$$\mathcal{H} = -\frac{1}{2}\ [\boldsymbol{B}(t) + \boldsymbol{K}(t)].\sigma$$

where $\langle K(\tau)K(0)\rangle_K = C^2\delta(\tau)$



Adiabatic evolution during one-time step

$$\left\langle \exp\left[\mathrm{i}|\mathbf{B}+\mathbf{K}|\delta t + \mathrm{i}\delta\phi\ \cos\theta_{(\mathbf{B}+\mathbf{K})}\right]\right\rangle_K$$

$$\langle\ldots\rangle_K = \int \mathrm{d}K(\ldots)\exp[-K^2\delta t/C^2]$$

$\langle\cos\theta_{(\mathbf{B}+\mathbf{K})}\rangle \Rightarrow$ **Modification of real (phase) geometric term**

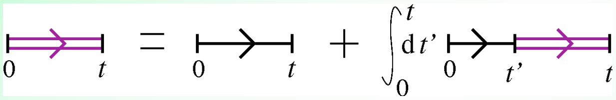
cross-terms in completed squ.

⇒ **Imaginary part of geometric term**

⇒ **geometric dephasing**

Quantum calculation: Master eqn

Real time Dyson eqn :



exact Master equation for spin's density matrix

Schoeller-Schon(1994)

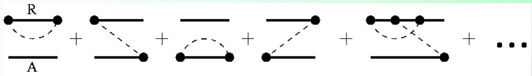
$$\frac{\mathrm{d}}{\mathrm{d}t}\ \rho_{ij}(t) = -\mathrm{i}\ [\mathcal{H}_{\mathrm{syst}},\rho\ (t)]_{ij}\ + \int_0^t \mathrm{d}\tau\ \ \Sigma_{ij,i'j'}(\tau)\ \rho_{i'j'}(t\text{-}\tau)$$

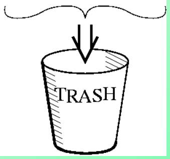
Sum of irredicible "self-energy" diagrams

APPROXIMATIONS : systematic weak-coupling and Markov

Bloch-Redfield (1957)

$$\Sigma_{ij,i'j'}(\tau) \ =$$





+ Assume environ. memory time, $t_{\mathrm{memory}} \ll T_2$

...but not $t_{\mathrm{memory}} \ll 1/B$

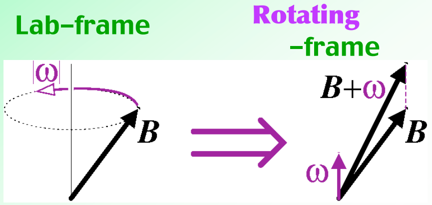
& no rotating wave approx

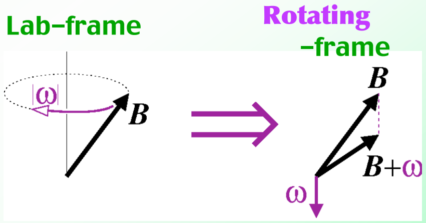
Berry phase is differential of **complex energy gap**;

$$\Phi_{\mathbf{Berry}} = \oint \mathrm{d}\varphi \, \frac{\mathrm{d}}{\mathrm{d}B_z} [B + \delta B + \mathrm{i}\,\Gamma_2]$$

$$\boxed{\Gamma_2 = T_2^{-1}}$$

Easily understood by going to **rotating frame**





$$\left.\begin{array}{r}\text{Lamb shift, } \delta B\\ \text{dephasing rate, } \Gamma_2\end{array}\right\} \text{Functions of rotating frame gap } \propto |B+\omega|$$

⇒ Taylor expand in ω

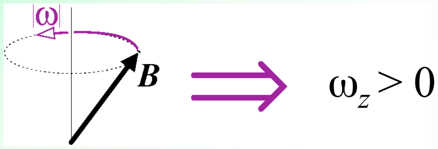
⇒ both have ω–terms ⇒ **Geometric terms**

Geometric dephasing

Imaginary part of Berry phase ⇒ dephasing

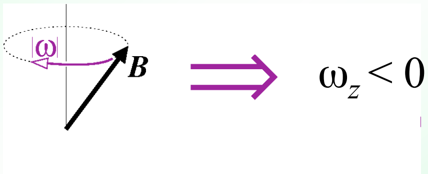
$$\mathrm{Im}[\Phi_{\mathbf{Berry}}] = \int_0^t \mathrm{d}t' \, \omega_z \frac{\mathrm{d}\Gamma_2}{\mathrm{d}B_z}$$

Can be **either sign**; depends of direction of winding



$\omega_z > 0$: geometric **dephasing** (positive)

⇒ **increase total dephasing**

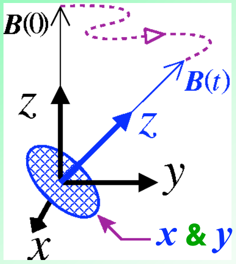


$\omega_z < 0$: geometric **REPHASING** (negative)

⇒ **REDUCES total dephasing**

...but it is only a **small** modification of total dephasing

Gauge-independence for open paths?



Ambiguity in choice of x & y axes
$\Rightarrow$ gauge-dependence
of Berry phase for open paths

x & y axes somewhere in this plane

Dephasing affects magnitude of off-diag. elements of density matrix

Off–diag. matrix element $= \langle\sigma_x\rangle \pm \mathrm{i}\langle\sigma_y\rangle$

Magnitude $= \sqrt{\langle\sigma_x\rangle^2 + \langle\sigma_y\rangle^2}$

Independent of choice of x & y axes $\Rightarrow$ gauge-independent

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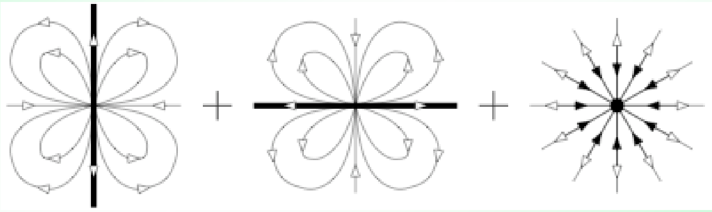
Isotropic enviroment: no modification of Berry phase & no geometric dephasing

Consider isotropic coupling to environment

Isotropic $\equiv$ z-axis coupling + y-axis coupling + x-axis coupling



All three couplings equal
➡ all three “quadrupoles” have equal strength

“Quadrupoles” sum to ZERO
➡ Berry phase unmodified by environment
➡ No geometric dephasing

Geometric (topological/holonomic) quantum computing

big problem

adiabatic manipulation is ***very slow***

one gate operation takes many coherent oscillations

cf. **dynamic manipulation:** 1gate operation ~ 1 coh. osc.

To reduce non-adiabatic effects to $< 10^{-4}$

each gate operation must take **time** $> 10^4$ coherent oscillations

... but

perhaps adiabatic manipulation has much less decoherence?

Needs decoherence time $> 10^4$ longer than dynamic manipulation

Does adiabatic manipulation have less decoherence?

Berry set-up : Environment decoheres adiabatic system at same rate as static system

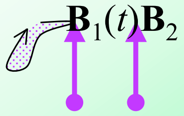
Reason: fluctuations of dynamic phase

bad!

Adiabatic manipulation with degenerate levels (non-abelian)

Appears better: no dynamic phase

$$|\psi_0\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$
$$|\psi_1\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$



... but "accidental" degeneracy – not respected by environment

If two spins see different environments:
average dyn. phase = 0, but fluctuations are as static system

If two spins see same environment via operator like $[\sigma_z^{(1)} + \sigma_z^{(2)}]$
average dynamic phase = 0, but fluctuations exist for $|\psi_1\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

not good!

Avoiding noise by symmetry

System-environment coupling = $\hat{O}_{\text{system}} \, \hat{O}_{\text{environ}}$

Barenco *et al* (1997)

Decoherence free subspaces : find symmetry of $\hat{O}_{\text{system}}$

i.e. two spins with $\hat{O}_{\text{system}} = [\sigma_z^{(1)} + \sigma_z^{(2)}]$

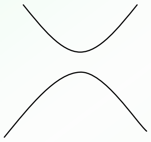
➡ **state** $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ **is decoupled from environment**

Two such states form *decoherence free subspace.*

... but another environment coupling will always "spoil the party"

Saclay "quantronium" : find symmetry of $\hat{O}_{\text{environ}}$

decoherence time = 10^4 **coherent oscillations**



Coupling = $\alpha\,(B_x, B_y, B_z)\,(a^\dagger + a)\,\sigma + \beta\,(B_x, B_y, B_z)\,(a^\dagger + a)^2\,\sigma$

Symmetry point where $\alpha\,(B_x, B_y, B_z) = 0$

Motto: can use symmetries to remove worst decoherence
...but don't expect no decoherence

Open questions for geometric quantum computing

(pure speculation)

- **avoid noise by symmetry**

  find system with a noise-"free" parameter space
  (symmetry surface not point)

- **use adiabatic manipulation in noise-"free" space**

Practical questions:

- **adiabatic manipulation is only best route**
  if dynamic manipulation impossible inside noise-"free" space.
  Could such a situation exist?

- **is decohence time in noise-"free" space**
  $> 10^4$ **longer than elsewhere?**

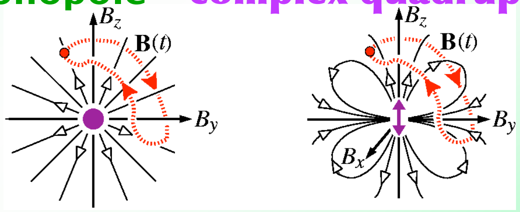
Summary of geometric phases with decoherence

❶ Geometric phases observable for **weak**-dissipation; $BT_2 >> 1$

i.e. small **matrix elements** for spin-flip

❷ Berry phase **modified** by **anisotropic** environment

For spin ➠ **monopole** + **complex quadrupole**



❸ **Geometric dephasing** : increases/**decreases** dephasing

❹ **quantum computing using adiabatic manipulation:**

No such thing as a free lunch!

a priori : worse than dyn. manipulation ... potential to improve??