Understanding quantum transport through chaotic systems

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Previously: Université de Genève

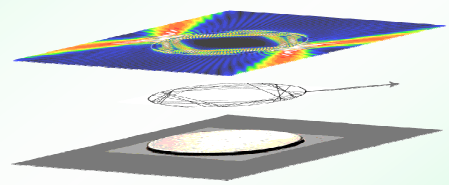
♣ R.W. & Ph. Jacquod, PRL **94**, 116801 (2005)
♣ Ph. Jacquod & R.W., PRB (2006) cond-mat/0512662
♣ R.W. & Ph. Jacquod, cond-mat/0512516

see also : **Heusler-Muller-Braun-Haake**, PRL **96**, 066804 (2006) & cond-mat/0511292
Rahav-Brouwer, cond-mat/0512095 & cond-mat/0512711

Avril 2006

Quantum chaos

chaotic lasing cavity



Stone's group Nature (2000)

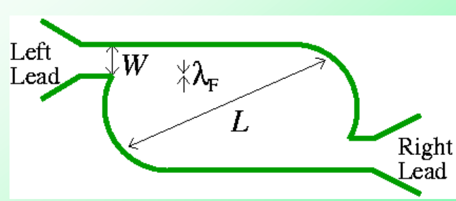
Quantum mechanics of a classically chaotic system.

Individual chaotic systems: unique

...but average properties: UNIVERSAL (often RMT)

AIM : understand universality

Average transport properties:



$$\lambda_F \ll W \ll L$$

semiclassical ↑ ↑ classically chaotic

Random matrix theory (RMT)

Quantum system with a random Hamiltonian.
Matrix elements are randomly chosen (but symmetric/Hermitian).
One parameter : width of gaussian distribution of elements

closed system : level-statistics (level-repulsion, GOE, GUE, etc)

open system :
- ♠ weak localisation (magnetoconductance) = -1/4
- ♠ universal conductance fluctuations = 1/8
- ♠ shot noise : Fano factor = 1/4

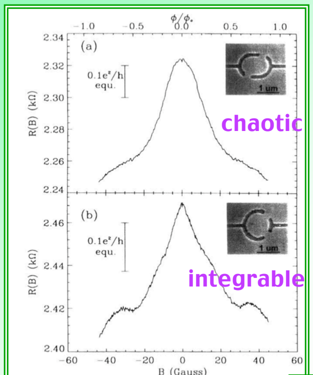
UNIVERSALITY: Quantum chaotic systems fit RMT

- ♠ spectrum of *N*-particles in nuclei
- ♠ spectrum of hydogen atom in strong *B*-field
- ♠ spectra of particles in many chaotic potentials

(Sinai billiard, stadium, etc)

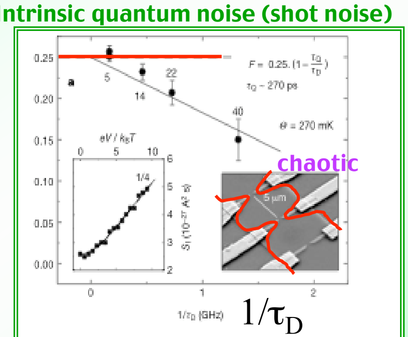
Transport measurements

♣ Magneto-conductance (weak localization)



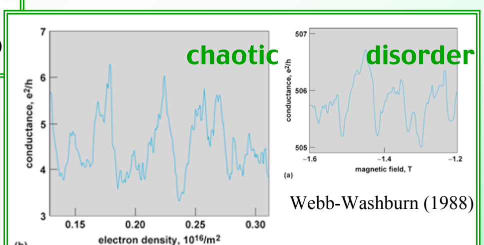
Chang *et al*, PRL **73**, 2111 (1994)

♣ Intrinsic quantum noise (shot noise)



S. Oberholzer *et al*, Nature **415**, 765 (2002)

♣ Universal conductance fluctuations



Webb-Washburn (1988)

Keller *et al*, Surf. Sci **305**, 501 (1994)

If we have:
(i) chaos
(ii) few short paths

then beautiful RMT properties?

... err, NO!!

Why does random matrix theory work so well ?

Why doesn't random matrix theory work ?

Ehrenfest time, t_E, a measure of classical-ness

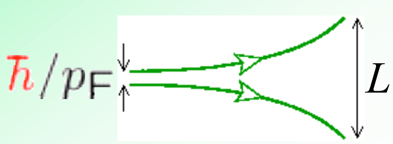
The Ehrenfest time, t_E .

Ehrenfest time = time for **minimal wavepacket** to spread over system's phase-space

Aleiner-Larkin, PRB **54**, 14423 (1996)

Wavepacket spreads under **classical chaotic flow**

if potential is smooth on scale of λ_F



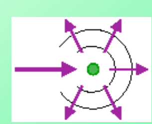
$$t_E = \lambda^{-1} \ln[p_F L/\hbar]$$

Lypunov exponent

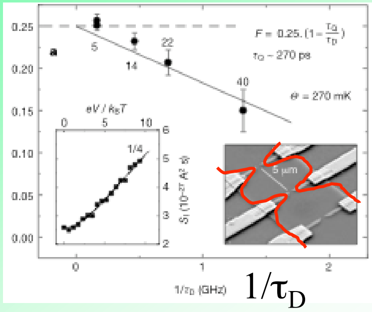
Cf. Potential with δ-correlated disorder:

wavepacket covers momentum-space after τ

and diffuses in position-space ➡ $t_E \sim t_{\text{Thouless}} \approx (L\tau/v_F)^{1/2}$



Ehrenfest time in expt and numerics



Shot noise experiment

Oberholzer *et al.* (2002)

Fano factor (ratio to Poissonian noise) **:**

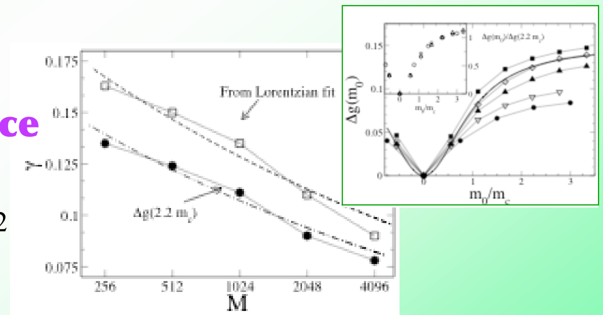
$$F \sim 1/4 \times \exp[- t_E/\tau_D]$$

Weak-localization dip in magneto-conductance

Numerics (kicked rotator map)

Ph. Jacquod & R.W cond-mat/0512662

$$g_{wl} \sim -1/4 \times \exp[- t_E/\tau_D]$$



See also Rahav-Brouwer, PRL **95**, 056806 (2005); cond-mat/0507035 (2005)

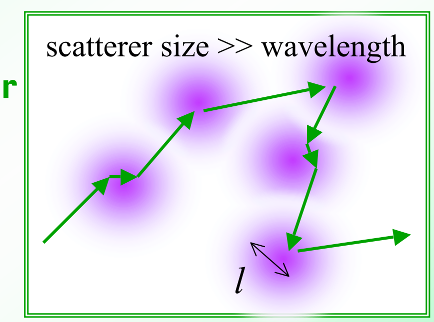
contradicts earlier numerics by Tworzydło *et al*, PRB **70**, 205324 (2004)

"Old" theory : quasiclassics with disorder

Diffusons/Cooperons for *s*-wave disorder: scatterer size < wavelength

Diffusons/Cooperons for smooth disorder

Aleiner-Larkin, PRB **54**, 14423 (1996),
Agam-Aleiner-Larkin, PRL **85**, 3153 (2000)
Rahav-Brouwer, PRL **95**, 056806 (2005)

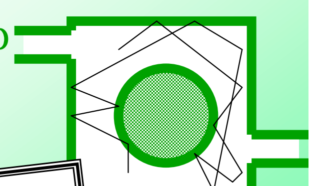


eqn. motion for "diffuson" :

$$\left[-i\,\omega + \mathcal{L} + \gamma\left(\partial/\partial\phi\right)^2\right] D(\omega;1,2) = \delta(1,2)$$

Model for clean chaotic systems?

Does give qualitatively correct answers. (universality?)



Price:

- ♠ introduce fictitious disorder ◂◂ UGLY!!
- ♠ wrong Ehrenfest time: $t_E \sim \ln[l/\lambda_F]$
- ♠ techically demanding
- ♠ breaks time-reversal symm...

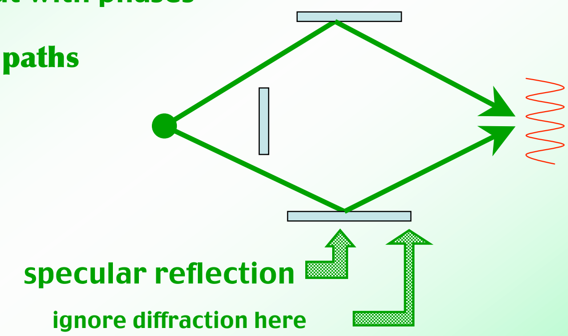
We want simpler model for clean system using only basic assumptions about classical chaos

Semiclassics = geometric optic

Particles follow classical paths but with phases

=> interference between paths

phase = classical action/hbar



wavelength < detector size << all other lengthscales

Use Landauer-Buttiker approach

=> need "geometric optics" for scattering matrix

=> need classical dynamics of chaotic system

Scattering matrix ⇒ transport properties

Scattering matrix: $S = \begin{pmatrix} r & t^\dagger \\ t & r' \end{pmatrix}$ (Landauer-Buttiker)

Transmission matrix : $\mathbf{T} \equiv t^\dagger t$

Dimensionless conductance : $g = \mathrm{tr}\,[\mathbf{T}] = \sum_{nm} |t_{nm}|^2$

Shot noise : quantum noise in DC current (at zero temperature)

$$S = \int \mathrm{d}t < I(t)I(0) - I^2 > \; = \; \mathrm{tr}\,[\mathbf{T}(1\text{-}\mathbf{T})]$$

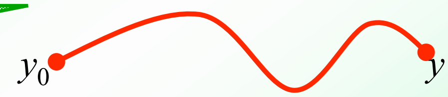
$$\text{Fano factor} = \frac{\mathrm{tr}\,[\mathbf{T}(1\text{-}\mathbf{T})]}{\mathrm{tr}\,[\mathbf{T}]} \propto \frac{\text{current noise}}{\text{average current (signal)}}$$

Semiclassics for scattering matrix

Energy Greens funct: $G(\mathrm{r},\mathrm{r}_0;E) = \sum_\gamma A_\gamma \exp[\mathrm{i}\, S_\gamma / \hbar]$

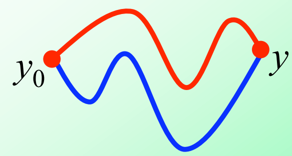
$A_\gamma^{\,2}$ = **classical stability of path** γ

Scattering matrix elements:



$$S_{nm} = (\mathrm{i}/h)^{1/2} \int \mathrm{d}y_0 \int \mathrm{d}y \sum_\gamma A_\gamma \exp[\mathrm{i}\, S_\gamma / h] \; \langle n|y\rangle \langle y_0|m\rangle$$

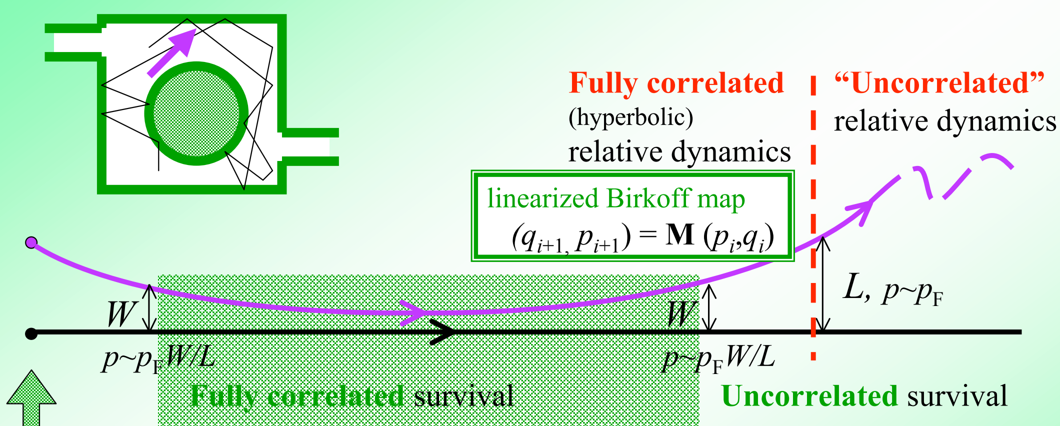
$$\sum_n \langle y|n\rangle\langle n|y'\rangle \approx \delta(y'-y)$$



$$\mathrm{tr}[\mathbf{t}^\dagger\mathbf{t}] = \sum_{nm} |t_{nm}|^2 = h^{-1} \int \mathrm{d}y_0 \int \mathrm{d}y \sum_{\gamma1,\gamma2} A_{\gamma1} A_{\gamma2} \exp[\mathrm{i}\,(S_{\gamma1} - S_{\gamma2})/ h]$$

Diagonal terms: $\sum_{\gamma1} A_{\gamma1}^{\,2}\, [\ldots]$ = **classical probability** $\times\, [\ldots]$

Classical dynamics in generic chaotic system



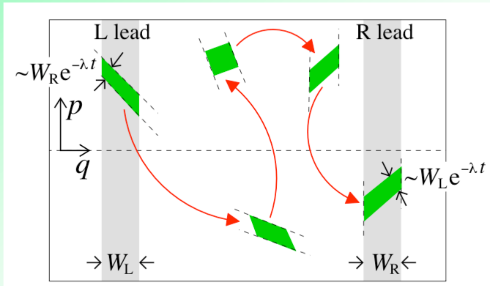
Here: If **black** path survives then so does purple path

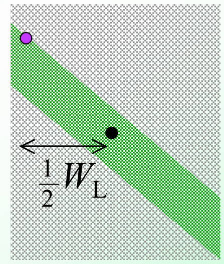
Elsewhere: *no* correlation of survival
even when relative dynamics are fully correlated

Paths at lead: paths from same lead have distance $q < W$, but if :
(i) $p \sim p_F$ then 2 paths initially **"uncorrelated"**
(ii) $p << p_F$ then 2 paths initially **fully correlated**

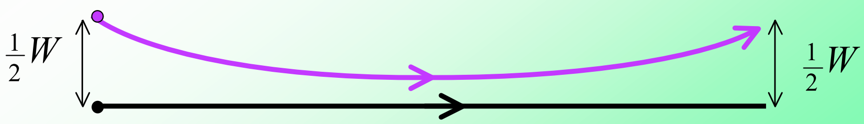
Classical dynamics in phase space

assume: hyperbolic flow at small scales
random flow on large scales



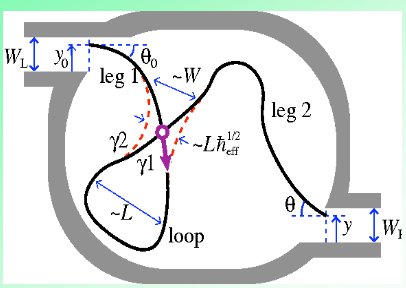


Unfolded hyperbolic dynamics within the band
(relative to black path)



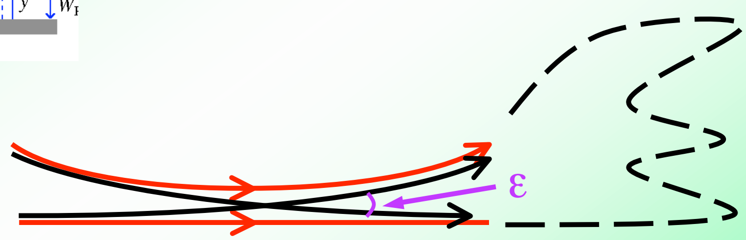
Weak Localization

Larkin-Khmelnikski (1982)
Richter-Sieber (2002)



♣ action difference $\delta S = \lambda^{-1} E_F\ \varepsilon^2$

♣ each path has time-reverse
with same action

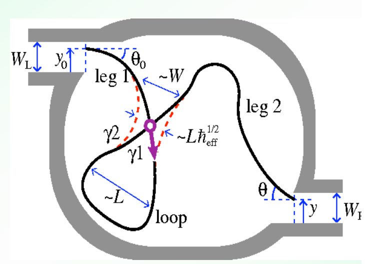


$A_{\gamma 2} \approx A_{\gamma 1}$ so $\sum_{\gamma 1} A_{\gamma 1}{}^2 [\ldots] =$ classical prob. $\times [\ldots]$

weak-loc = classical prob. of loop $\times \exp[\, i$ action diff.$]$

Classical probability of crossing

$\propto$ (phase–space area)$^{-1}$ x $\varepsilon d\varepsilon$ x $dt\ dt_{loop}\ dt_{leg1}$



Uniform escape rate: $\exp[-t/\tau_D]$ for time of path t.

♣ Minimum time for

loop : $t^{loop} = 2\lambda^{-1}\ln[\varepsilon]$ leg : $t^{leg} = \lambda^{-1}\ln[\varepsilon^{-1}(W/L)]$

♣ reduced escape probability within $t^{leg} = \lambda^{-1}\ln[\varepsilon^{-1}(W/L)]$ of crossing

$$\int_{t^{loop}+2t^{leg}}^{\infty} dt\ \exp[-(t-2t^{leg})/\tau_D]\ (t-(t^{loop}+2t^{leg}))^2 \quad \Rightarrow \quad \exp[-2t^{leg}/\tau_D] = \varepsilon^{2/(\lambda\tau_D)}$$

Integral over crossing angles, ε :

(phase–space area)$^{-1}$ x $\mathrm{Re}\int \varepsilon d\varepsilon\ \varepsilon^{2/(\lambda\tau_D)} \exp[\,iE_F\ \varepsilon^2/\lambda h]$ => N^{-1}

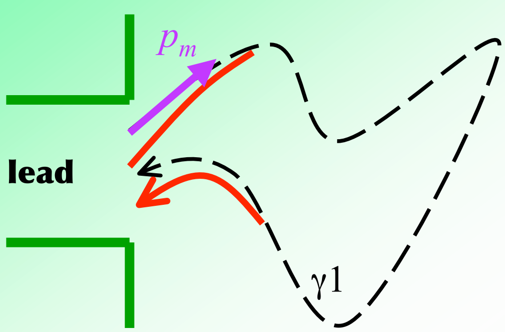
Is zero without $\varepsilon^{2/(\lambda\tau_D)}$ everything comes from finite length of legs/loop

$g_{wl} \sim$ Drude conductivity x N^{-1}

Result: $g_{wl} = -1/4 \times \exp[-t_E^{cl}/\tau_D]$

due to paths > $(t_E^{cl} + t_E^{op})$

Coherent back-scattering peak



Baranger-DiVincenzo-Jalabert-Stone (1991)
Richter-Sieber (2002)

♣ diagonal contribution

γ2 = exact time–reverse of γ1

♣ actions of γ2 & γ1 equal

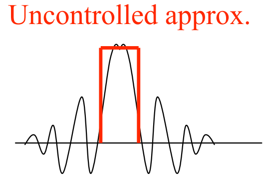
$|r_{nm}|^2_{coh.\ back.} = \delta_{nm}$ x classical prob. to return to mode m, $(\,p = p_m \pm h/W\,)$

Result: $R_{cbs} = 1/2$ independent of (t_E^{cl}/τ_D)

... then transmission is t_E^{cl}-dependent

& reflection is t_E^{cl}-independent

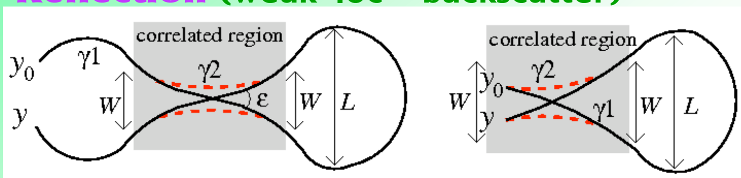
Uncontrolled approx.



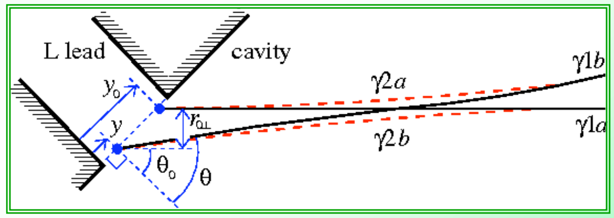
Current not conserved! scattering matrix not unitary!!

Coherent back-scattering peak

Reflection (weak–loc + backscatter)



$R_{wl} = -1/4 \times \exp[-t_E^{cl}/\tau_D]$



INGREDIENTS as before

♣ δS and lengths in terms of $(r_{0\perp}, p_{0\perp})$ instead of ε

Result: $R_{cbs} = 1/2 \times \exp[-t_E^{cl}/\tau_D]$ due to paths > $(t_E^{cl} + t_E^{op})$

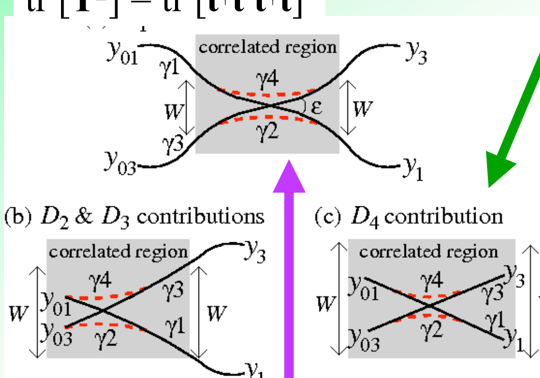
Current conserved ✓ scattering matrix unitary ✓

Shot Noise

$$\text{Fano factor} = \frac{\text{tr}\,[\mathbf{T}(1-\mathbf{T})]}{\text{tr}\,[\mathbf{T}]}$$

tr [**T**] = tr [**t**†**t**]
= Drude cond. (γ2 = γ1)

tr [**T**²] = tr [**t**†**t t**†**t**]



For times < t_E^{op} **: contributions to** tr [**T**²]
cancel contributions to tr[**T**]

No contribution to shot noise from paths < t_E^{op}

Result:
$F = 1/4 \times \exp[-t_E^{op}/\tau_D]$

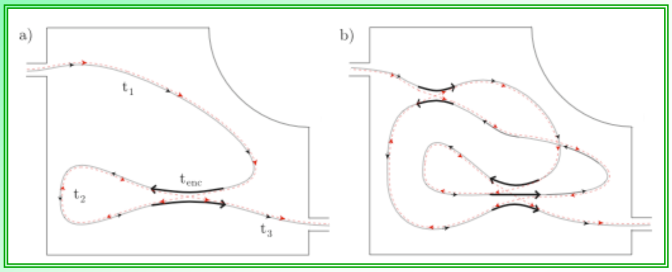
For times < t_E^{op} **:**

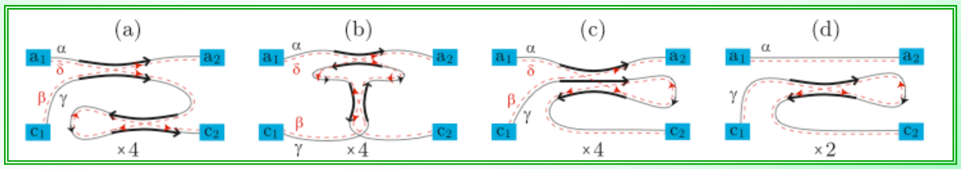
contributions D_4 => *RMT result* =1/4 for paths > t_E^{op}

contributions $D_{2,3}$ to tr [**T**²] **cancel** tr[**T**]

Summing to all orders in 1/N

Heusler–Muller–Braun–Haake, PRL **96**, 066804 (2006) & cond-mat/0511292

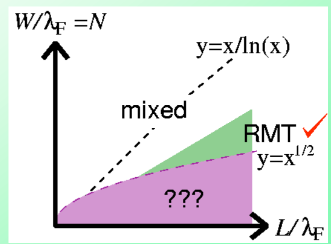




Prove RMT results to all orders in N^{-1}

But only valid for $0 < t_E << t_D$

i.e. $(\lambda_F L)^{1/2} < W << L \ln (L/\lambda_F)$



Conclusions

Trajectories shorter than Ehrenfest time: "Classical" contributions

(i) noiseless
(ii) no interference effects
(iii) separate subspace in scattering matrix (subject of another talk)

Trajectories longer than Ehrenfest time: "Quantum" contributions

(i) random matrix theory (RMT) shot noise
(ii) suppressed weak localization & coherent backscattering
goes to RMT only in limit $t_E << \tau_D$
(iii) possibly RMT conductance fluctuations (Brouwer-Rahav)

Cavity behaves like two cavities (two fluids)

one quantum & one classical
... but quantum fluid not RMT

(proves 2-fluid model *but* invalidates effective RMT model)