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Noise of *quantum* chaotic systems in the *classical* limit

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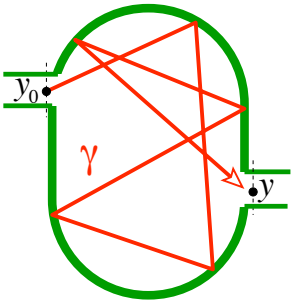
R.W. Phys. Rev. B **75** **235404 (2007)**
R.W. and Ph. Jacquod, Phys. Rev. Lett. **96** 206804 (2006)
Ph. Jacquod and R.W. Phys. Rev. B **73** 195115 (2006)
R.W. and Ph. Jacquod, Phys. Rev. Lett. **94** 116801 (2005)

OUTLINE



Quantum particles: Noise in current

Classical particles: No noise in current

...and *classical* limit of *quantum*?

Random matrix theory (RMT)

$\Rightarrow$ *"noise in classical limit"* X

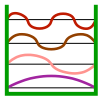
NEW REGIME **in classical limit — Not RMT**

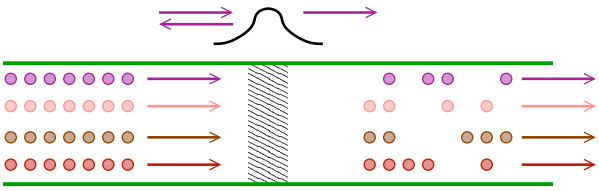
Should revisit old questions for new regime:

Tunnel-barriers **on leads**

NOISE DUE TO TUNNEL-BARRIER

Multi-mode quantum wire:





Independent processes $\Rightarrow$ **central limit theorum**

$\langle \delta I^2 \rangle = F \times e \times I$

Fano factor,
$F = (1 - \text{transmission prob.})$

Classical limit: wavelength $\lambda_F \ll$ other scales

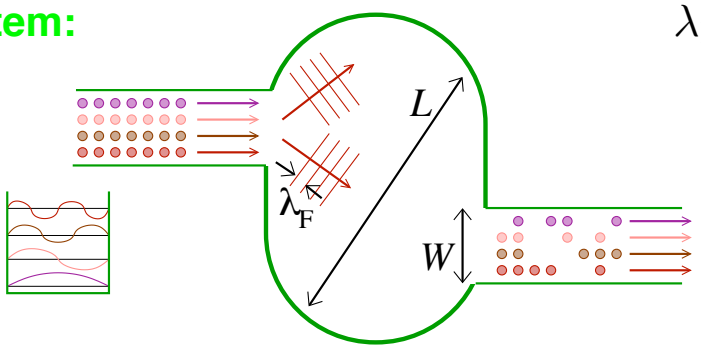
barriers $\Rightarrow$ *impenetrable/transparent* $\Rightarrow$ **noiseless**

CLASSICAL DETERMINISM

NOISE DUE TO CHAOTIC SYSTEM

Chaotic system:

$\lambda_F \ll W \ll L$



Bohigas-Giannoni-Schmit (1984): **obeys** *random matrix theory??*

$\langle \delta I^2 \rangle = F \times e \times I$

Fano factor, $F = 1/4$

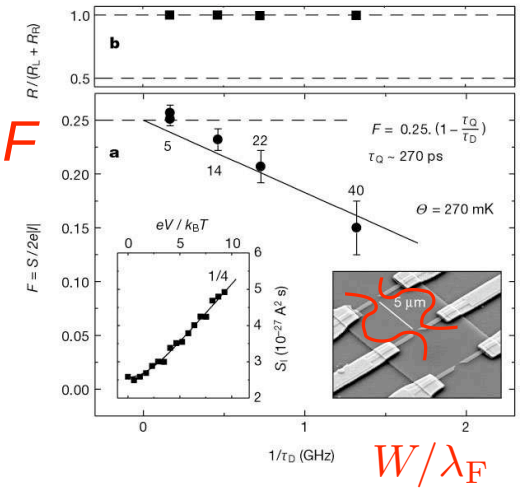
Classical limit ($\lambda_F \ll W, L$) : noise in I remains

NO CLASSICAL DETERMINISM

...similar for integrable system (i.e. rectangle): not RMT, but *expect* noise

EXPERIMENT SAYS "NOT RMT"

Oberholtzer et al, Nature (2002)

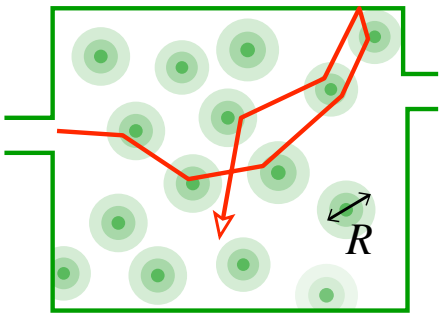


$$\langle \delta I^2 \rangle = F \times eI$$

...fits theory for *smooth disorder*

Aleiner-Larkin (1996,1997)

Agam-Aleiner-Larkin (2000)



$$\lambda_F \ll R$$

...introduced Ehrenfest time

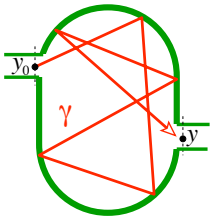
RAY-OPTICS for the 21st century

Landauer-Büttiker:

scattering matrix $\Rightarrow$ **Fano-factor**

$$\mathcal{S} = \begin{pmatrix} \mathbf{r} & \mathbf{t} \\ \mathbf{t}' & \mathbf{r}' \end{pmatrix} \qquad F = \frac{\mathrm{tr}\left[\mathbf{t}^\dagger\mathbf{t} - \mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}\right]}{\mathrm{tr}\left[\mathbf{t}^\dagger\mathbf{t}\right]}$$

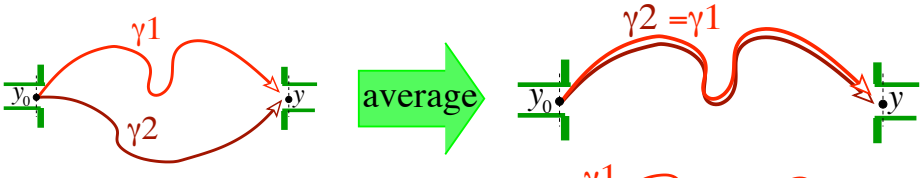
semiclassics *(vanVleck/Gutzwiller)*: $\sum$ over ***classical*** paths from mode m to n



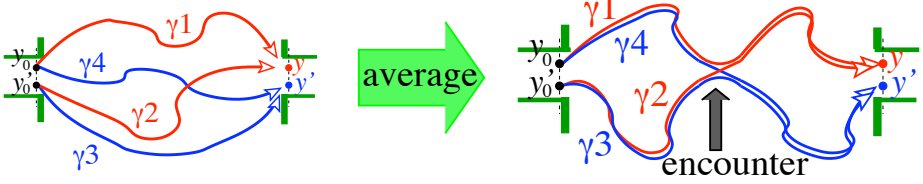
$$\mathbf{t}_{nm} = \sum_\gamma \mathcal{B}_{nm} \times A_\gamma \exp\left[\mathrm{i}S_\gamma/\hbar\right]$$

classical action/stability $\Rightarrow S_\gamma, A_\gamma$ join lead-modes $\Rightarrow \mathcal{B}_{nm}$

$\mathrm{tr}[\mathbf{t}^\dagger\mathbf{t}] =$

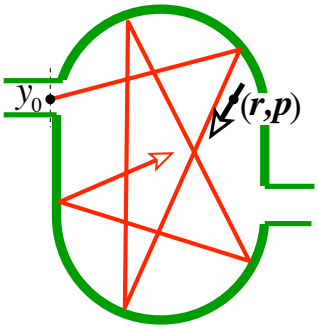


$\mathrm{tr}[\mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}] =$

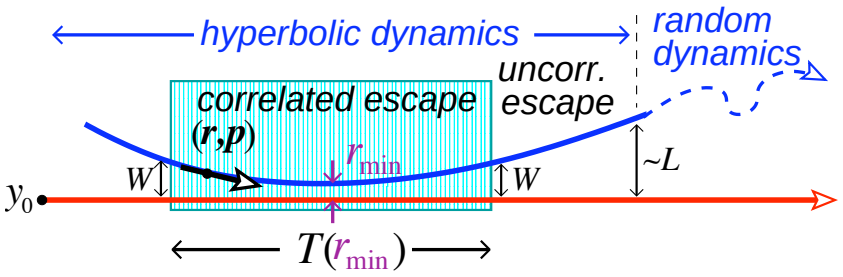


DYNAMICS OF CLASSICAL PATHS

Relative dynamics of paths



hyperbolic chaos: Lyapunov exponent = λ



Log. timescale: $T(r_{\rm min}) = 2\lambda^{-1}\ln[W/r_{\rm min}]$

Semiclassics: **Integrate over** $\exp[{\rm i}\delta S(r_{\rm min})/\hbar]$

$\Rightarrow$ **Encounter size** $\sim\sqrt{\lambda_{\rm F}}$

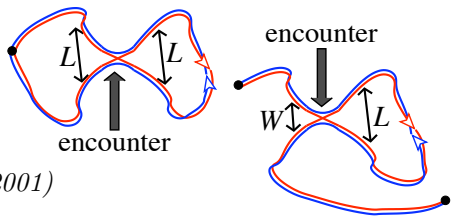
$\Rightarrow T(r_{\rm min}) \sim$ **Ehrenfest time** $=\lambda^{-1}\ln[L/\lambda_{\rm F}]$

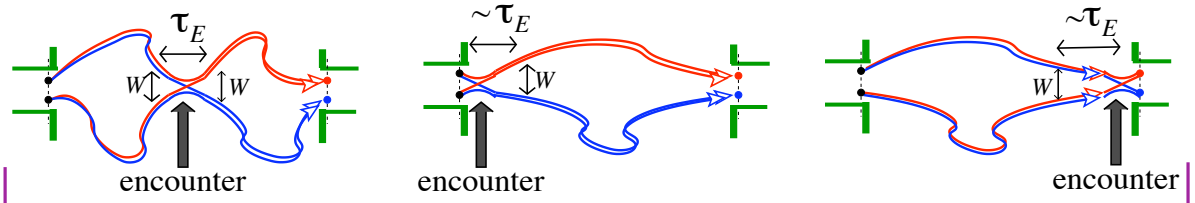
CALCULATING THE NOISE

Closed system: RMT level-statistics *Sieber(2001)*

$\Rightarrow$ non-perturb. *Haake group (2006)*

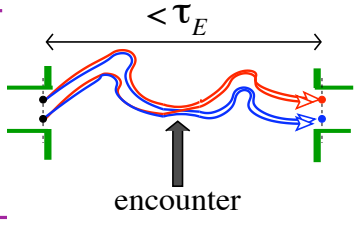
Open system: RMT weak-localization *Richter-Sieber(2001)*





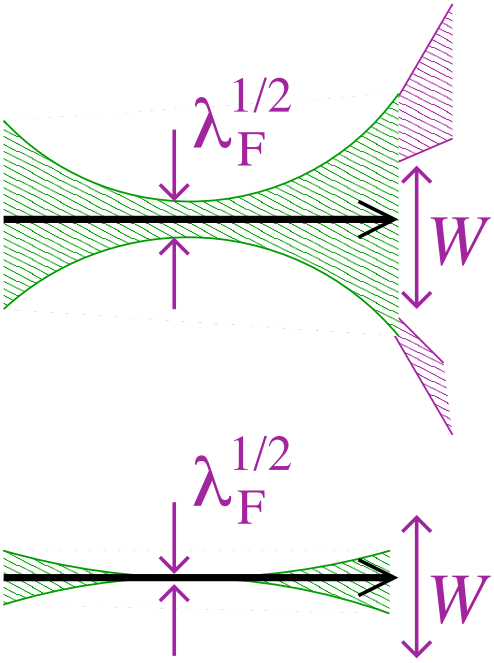
$\Rightarrow$ ***RMT*** **result**

classical **limit**



Braun-Heusler-Müller-Haake (2006), Whitney-Jacquod (2006)

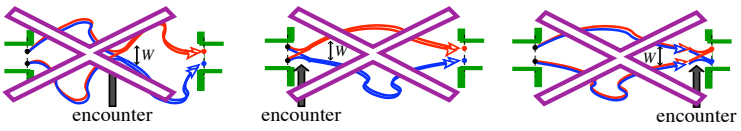
Hand-waving argument for noise

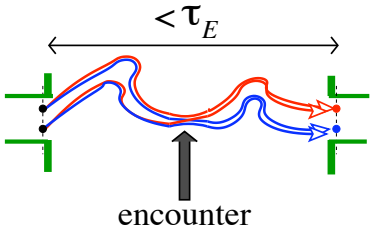


wavepacket escapes *in pieces*
$\Rightarrow$ *RMT NOISE*

wavepacket escape *as whole*
$\Rightarrow$ *NO NOISE*

Suppression in classical limit





Classical limit:
escapes *without* diverging to $\simeq W$

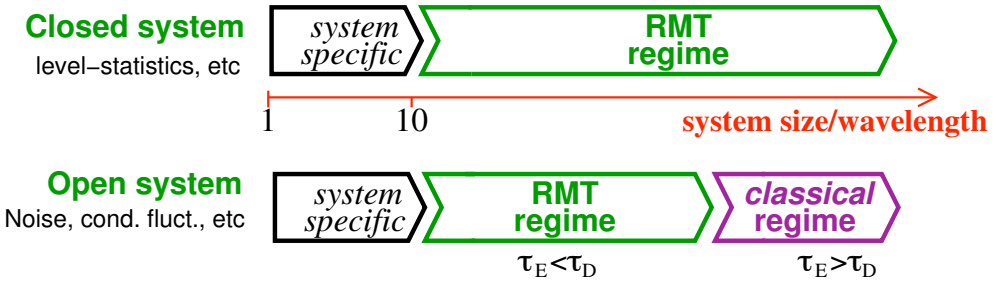
$$F \propto \mathrm{tr}\left[\mathbf{t}^\dagger\mathbf{t} - \mathbf{t}^\dagger\mathbf{t}\mathbf{t}^\dagger\mathbf{t}\right] \rightarrow \text{ zero}$$

Recover *CLASSICAL DETERMINISM*

Classical limit noiseless $\Rightarrow$ *not* random matrix theory (RMT)

proposed in Beenakker, van Houten (1991)

NEW CLASSICAL REGIME



Dwell time, $\tau_{\rm D}$, is time to escape system

Weight of *non*-classical contributions

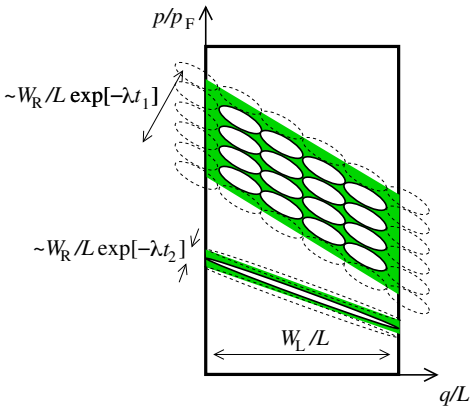
= probability to escape before Ehrenfest time, $\tau_{\rm E}$

= $\exp[-\tau_{\rm E}//\tau_{\rm D}]$

$\Rightarrow$ RMT-to-CLASSICAL cross-over *powerlaw* in $L/\lambda_{\rm F}$,
exponent= $(\lambda\tau_{\rm D})^{-1} \ll 1$

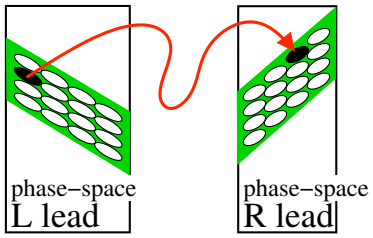
PHASE-SPACE (PS) BASIS

- complete & orthonormal basis: states *localized* in ***r*** and ***p***



States on *large* bands (area $> \hbar$)

1-to-1: ingoing $\leftrightarrow$ outgoing



Classical limit: DIAGONALIZE scattering matrix

ALL eigenvalues = 0,1

ALL cummulants of noise = 0

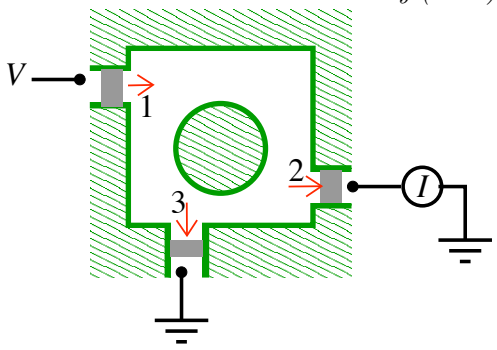
TUNNEL-BARRIERS ON LEADS

system $\Rightarrow$ ***classical limit***
barriers $\Rightarrow$ ***quantum***

tunnel probability $= \rho_j$ **for lead** j

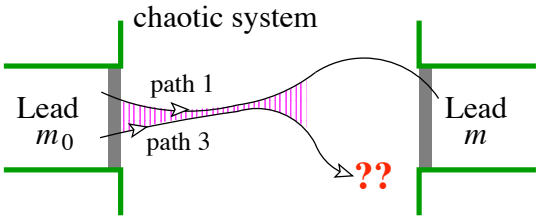
PS-basis not useful $\Rightarrow$ **usual semiclassics**

Whitney (2007)

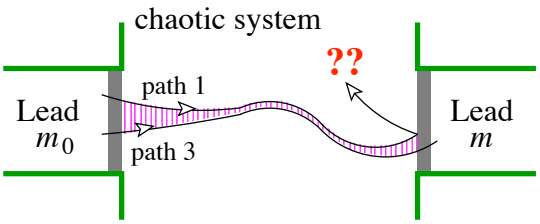


[A] a quantum contribution

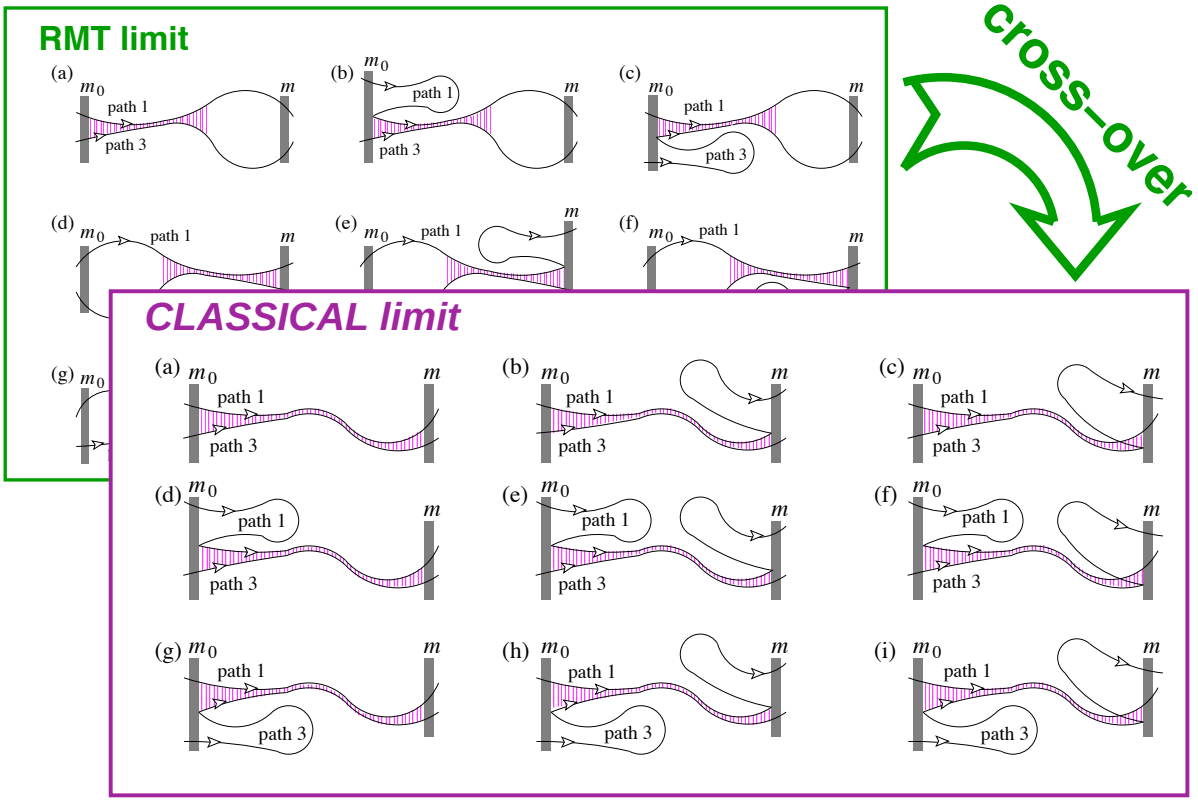
$\Rightarrow$ Random matrix result



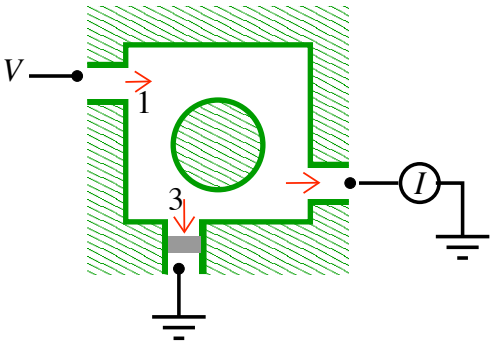
[B] a *classical* contribution

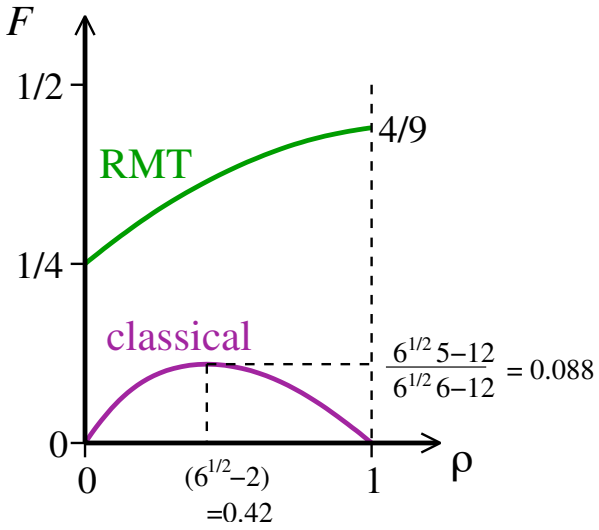


Exhaustive list of contributions



EXAMPLE: Tunnel-barrier on third lead

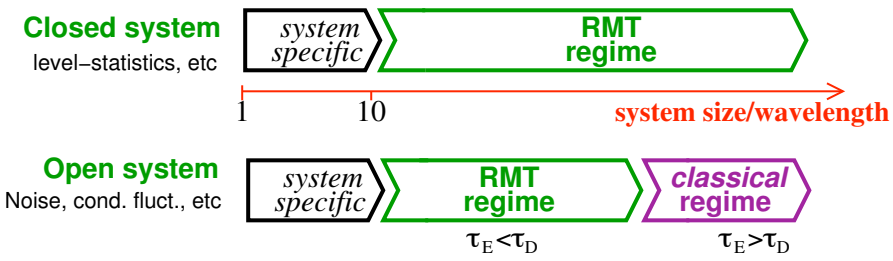




CONCLUSIONS

Noiseless transport in *classical* limit

- "wavepacket" escapes as a ***whole*** before spreading to lead width



New regime — new behaviour

- **Conductance fluctuations**

  =Lorentzian not $\left(\text{Lorentzian}\right)^2$

  Brouwer-Rahav in prep.

- **Tunnel barriers**

  $\Rightarrow$ non-monotonic noise