Understanding quantum transport through chaotic systems

"Supression of interference & shot noise *without* decoherence"

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Previously: Université de Genève

- ♣ R.W. & Ph. Jacquod, PRL **94**, 116801 (2005)
- ♣ Ph. Jacquod & R.W., PRB (2006) cond-mat/0512662
- ♣ R.W. & Ph. Jacquod, cond-mat/0512516

see also : **Heusler-Muller-Braun-Haake**, PRL **96**, 066804 (2006) & cond-mat/0511292
Rahav-Brouwer, cond-mat/0512095 & cond-mat/0512711

Max Planck (Dresden) May 2006

Quantum chaos : Quantum mechanics of a classically chaotic system.

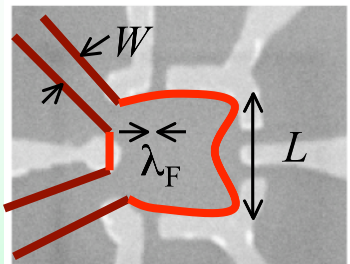
Individual systems: unique ... but average properties: UNIVERSAL

AIM : microscopic model of universality both Random matrix theory (RMT) and beyond

Transport properties through nanoscale chaotic system:

$$\lambda_F << W << L$$

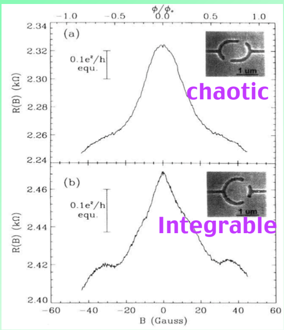
semiclassical ⇧ ⇧ "lots" of chaos



DiCarlo-Marcus-Harris(2003)

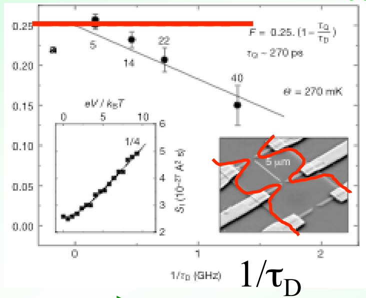
Transport properties: "easy" to measure in experiment

Magneto-conductance Expt.
weak localization



Chang *et al*, PRL **73**, 2111 (1994)

Shot noise Expt: quantum noise in DC current at zero temperature



S. Oberholzer *et al*, Nature **415**, 765 (2002)

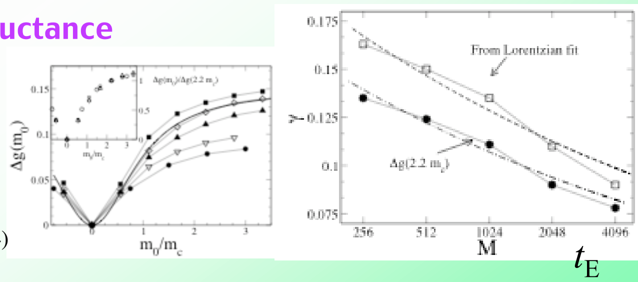
Chaos & few short paths ⇨ beautiful RMT properties?

... well, NO!!

NUMERICS for Magneto-conductance
(kicked rotator map)

Ph. Jacquod & R.W cond-mat/0512662

or Rahav-Brouwer, PRL **95**, 056806 (2005); cond-mat/0507035
contradicts earlier numerics
by Tworzydło *et al*, PRB **70**, 205324 (2004)



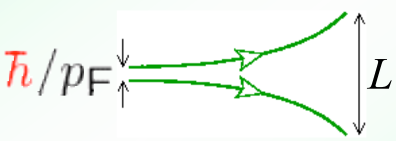
Shot noise and weak-localization decay "exponentially" with t_E/τ_D

Why does random matrix theory work ?
Why does random matrix theory fail ?

Ehrenfest time, t_E : a measure of **classical-ness**

t_E = time for **minimal wavepacket** to spread to **classical** scale

Spreads under **classical chaotic** dynamics



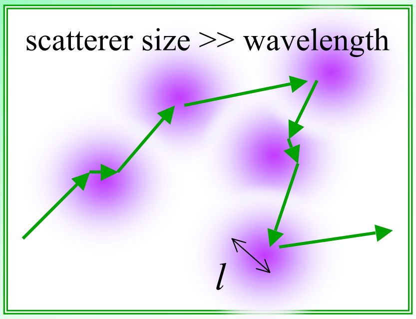
$$t_E = \lambda^{-1} \ln [p_F L/\hbar]$$

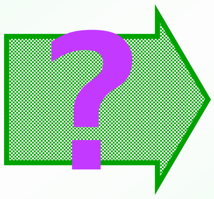
Lyapunov exponent

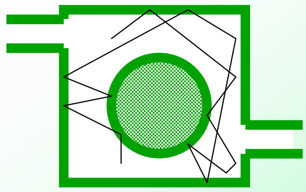
We assume no short range disorder : cf. Oleg Bulashenko talk

"Old" theory : quasiclassics with disorder

Diffusons/Cooperons for smooth disorder:







Aleiner-Larkin, PRB **54**, 14423 (1996),
Agam-Aleiner-Larkin, PRL **85**, 3153 (2000)
Rahav-Brouwer, PRL **95**, 056806 (2005)
Tian-Altland-Brouwer, cond-mat/0605051

Qualitatively fit to data for clean chaotic

We want simpler model for clean system using only basic assumptions about classical chaos

See also new ballistic sigma-model : Altland's group

Semiclassics ⇒ Scattering matrix ⇒ transport properties

Landauer–Buttiker scattering matrix:

$$\mathbf{S} = \begin{pmatrix} \mathbf{r} & \mathbf{t}^\dagger \\ \mathbf{t} & \mathbf{r}' \end{pmatrix}$$

CONDUCTANCE : $g = \langle \mathrm{tr}[\mathbf{t}^\dagger \mathbf{t}] \rangle$

SHOT NOISE : quantum noise in DC current (at zero temperature)

Fano factor: $F =$ "noise/current"

$$= g^{-1} \langle \mathrm{tr}[\mathbf{t}^\dagger\mathbf{t} - \mathbf{t}^\dagger\mathbf{t}\,\mathbf{t}^\dagger\mathbf{t}] \rangle$$

Semiclassical propagator ⇒ Scattering matrix:

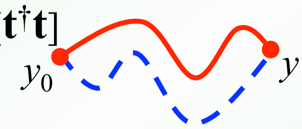
Lead mode wavefunctions

$$\mathbf{t}_{nm} = (i/h)^{1/2} \int dy_0 \int dy \; <n|y> <y_0|m> \; \sum_\gamma \; A_\gamma \exp[i\, S_\gamma / h]$$

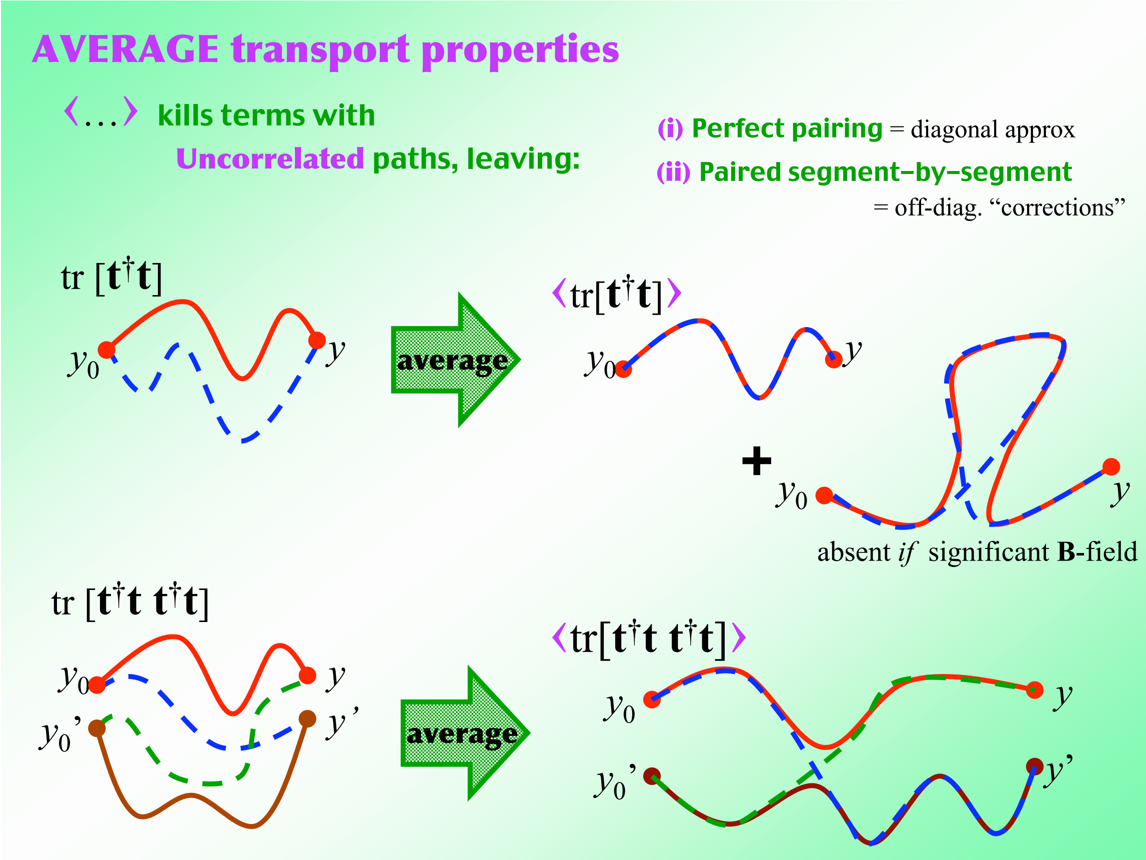
Van Vleck/Gutzwiller propagator = geometric optics

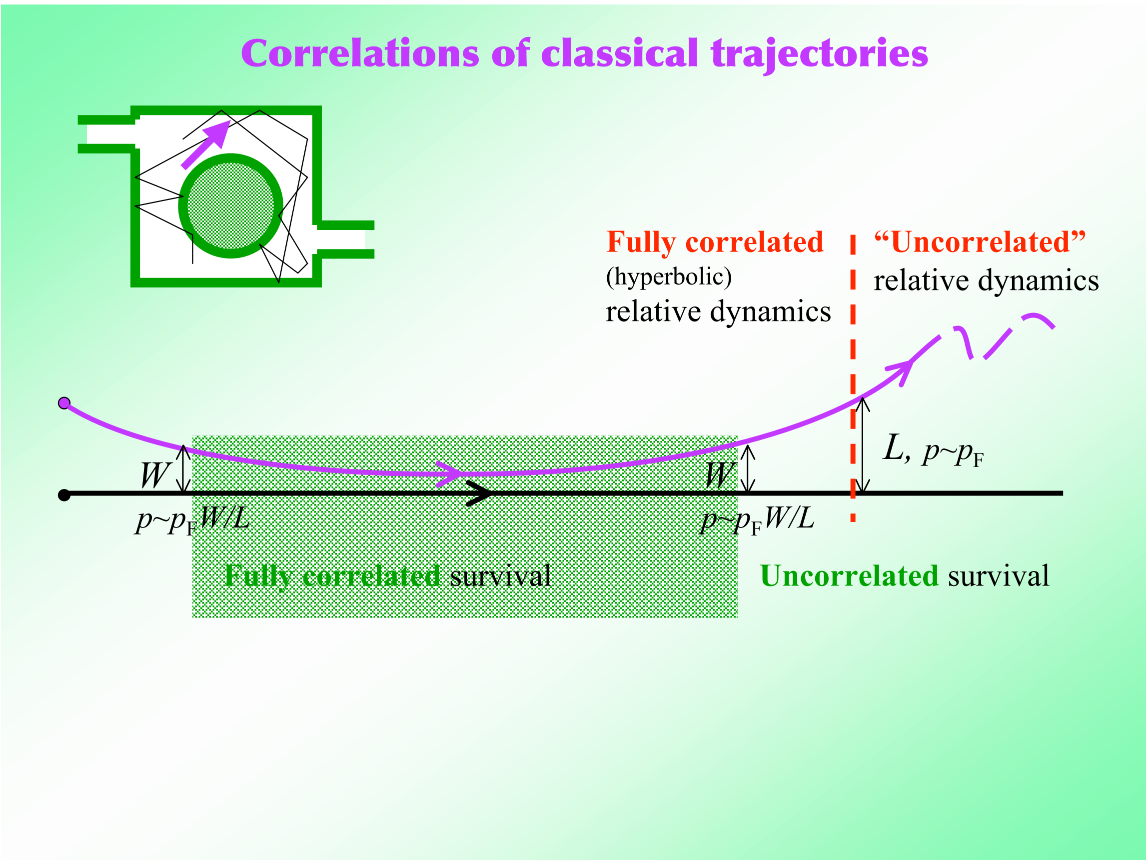
Sum over all classical paths: S_γ = classical action

tr $[\mathbf{t}^\dagger\mathbf{t}]$



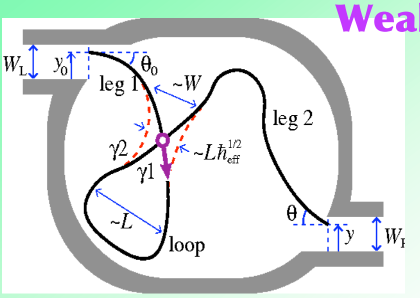
$\langle \ldots \rangle$ supresses terms with Uncorrelated paths
because $\langle \exp[i(S_{\gamma 1} - S_{\gamma 2})/h] \rangle$ killed by oscillation inside $\langle \ldots \rangle$


AVERAGE transport properties
⟨...⟩ kills terms with
Uncorrelated paths, leaving:
(i) Perfect pairing = diagonal approx
(ii) Paired segment–by–segment
= off-diag. "corrections"
tr [$\mathbf{t}^\dagger\mathbf{t}$]
y_0
y
average
⟨tr[$\mathbf{t}^\dagger\mathbf{t}$]⟩
y_0
y
+
y_0
y
absent *if* significant **B**-field
tr [$\mathbf{t}^\dagger\mathbf{t}\ \mathbf{t}^\dagger\mathbf{t}$]
y_0
y
y_0'
y'
average
⟨tr[$\mathbf{t}^\dagger\mathbf{t}\ \mathbf{t}^\dagger\mathbf{t}$]⟩
y_0
y
y_0'
y'

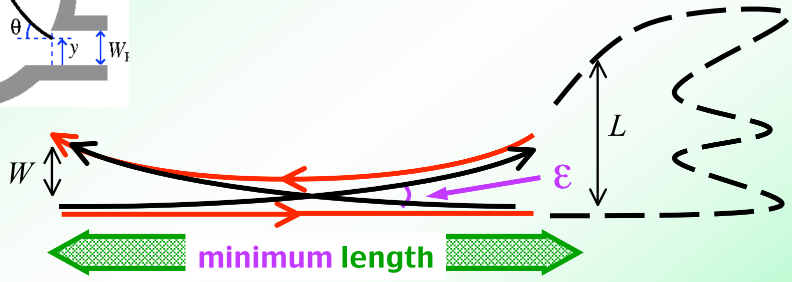

Correlations of classical trajectories
Fully correlated
(hyperbolic)
relative dynamics
"Uncorrelated"
relative dynamics
W
$p \sim p_F W/L$
W
$p \sim p_F W/L$
L, $p \sim p_F$
Fully correlated survival
Uncorrelated survival

Weak Localization

Larkin-Khmelnikski (1982)
Richter-Sieber (2002)



action difference between
red and black path, $\delta S = \lambda^{-1} E_F\, \varepsilon^2$



weak–loc = $\int d\varepsilon$ classical prob. of such path × exp[i $\delta S/h$]

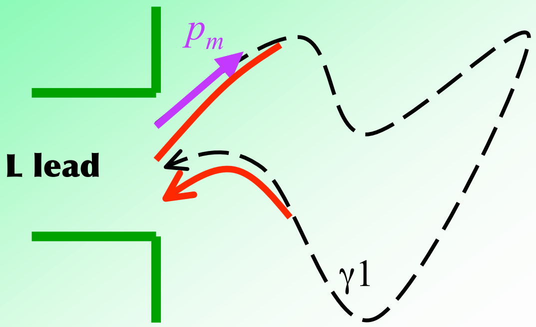
No contribution *if* less than 2 Ehrenfest times $(t_E^{op}+t_E^{cl})$

Complication: ε–integral is zero
except for finite length of legs/loop

Result: $g_{wl} = -1/4 \times \exp[-t_E^{cl}/\tau_D]$
$= \text{RMT result} \times \exp[-t_E^{cl}/\tau_D]$

Coherent back-scattering peak

Baranger-DiVincenzo-Jalabert-Stone (1991)
Richter-Sieber (2002)



♣ diagonal contribution
exact time–reverse of same path

Result: $R_{cbs} = 1/2$ independent of t_E

... but transmission is t_E-dependent

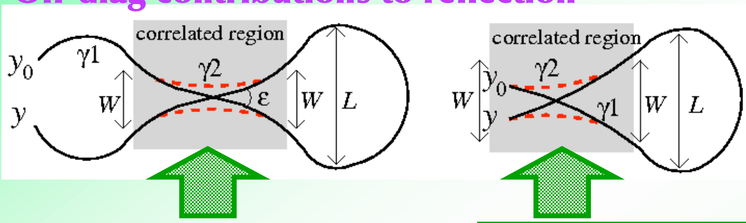
Current not conserved! scattering matrix not unitary!!

...BUT correlation of black and red paths ignored

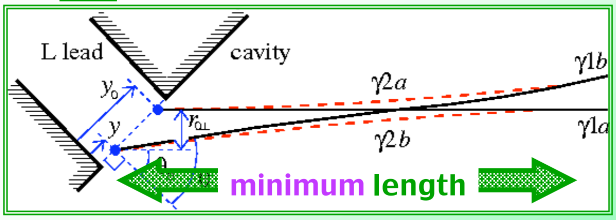
Coherent back-scattering peak

Sum over all lead modes first (unlike Rahav-Brouwer, cond-mat/0512711)

Off-diag contributions to reflection



$R_{wl} = -1/4 \times \exp[-t_E^{cl}/\tau_D]$



write δS and lengths in terms of $(r_{0\perp}, p_{0\perp})$

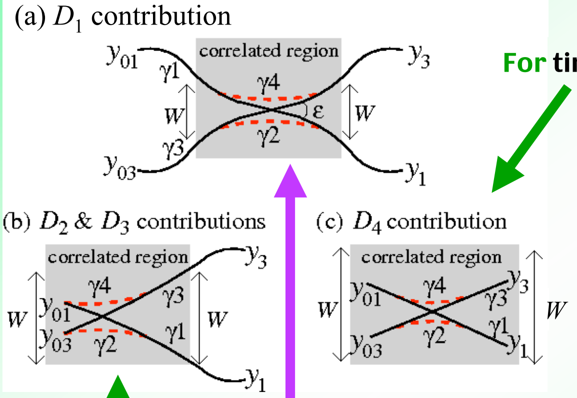
Result: $R_{cbs} = 1/2 \times \exp[-t_E^{cl}/\tau_D]$ due to paths > $(t_E^{cl} + t_E^{op})$

Current conserved ✓ scattering matrix unitary ✓

Shot Noise

$$\text{Fano factor} = \frac{\mathrm{tr}\,[\mathbf{t}^\dagger\mathbf{t}(1-\mathbf{t}^\dagger\mathbf{t})]}{\mathrm{tr}\,[\mathbf{t}^\dagger\mathbf{t}]}$$

Contributions to $\mathrm{tr}\,[\mathbf{t}^\dagger\mathbf{t}\;\mathbf{t}^\dagger\mathbf{t}]$



For times < t_E^{op} : D_4 cancels $\mathrm{tr}[\mathbf{t}^\dagger\mathbf{t}]$
All other contribution = 0

No contribution to shot noise from paths < t_E^{op}

Result:
$F = 1/4 \times \exp[-t_E^{op}/\tau_D]$
$= \text{RMT result} \times \exp[-t_E^{op}/\tau_D]$

For times > t_E^{op} :
contrib. D_1 => *RMT result* =1/4 for paths > t_E^{op}
& contrib. $D_{2,3}$ cancel $\mathrm{tr}[\mathbf{t}^\dagger\mathbf{t}]$

Conclusions

"Classical" contributions: trajectories < t_E

(i) noiseless

(ii) no interference effects (weak-localization, etc)

(iii) separate subspace in scattering matrix

Another talk: diagonalise *classical modes* of S matrix by using phase space basis

"Quantum" contributions (hierarchy of behaviours) :

(a) $t_E <$ trajectories $< 2t_E$

(i) RMT quantum noise

(ii) No weak localization or coherent backscattering

(b) trajectories $> 2t_E$

(i) RMT quantum noise

(ii) RMT weak localization & coherent backscattering

Summing all contributions:

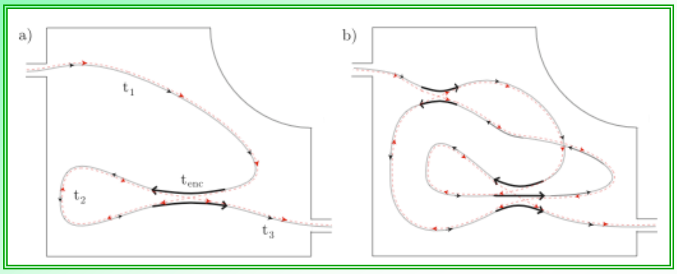
$$g_{wl} = \text{RMT result} \times \exp[-t_E^{cl}/\tau_D]$$

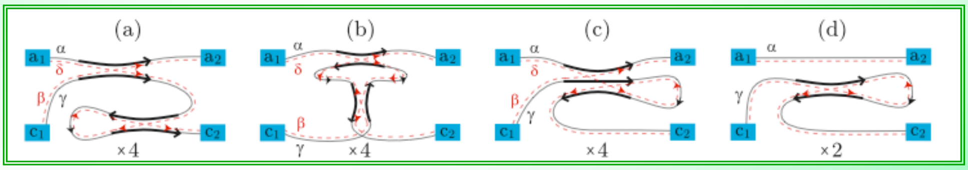
$$F = \text{RMT result} \times \exp[-t_E^{op}/\tau_D]$$

Conductance fluctuations? Tian-Altland-Brouwer, cond-mat/lastweek

Summing to all orders in 1/N

Heusler-Muller-Braun-Haake, PRL **96**, 066804 (2006) & cond-mat/0511292

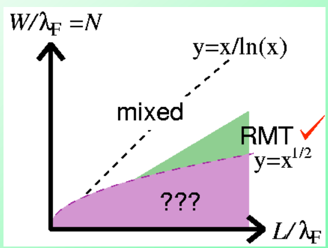




Prove RMT results to all orders in N^{-1}

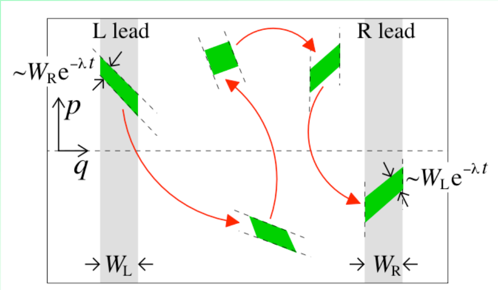
But only valid for $0 < t_E << t_D$

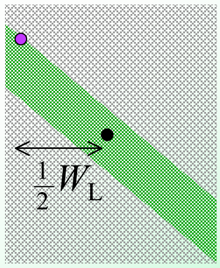
i.e. $(\lambda_F L)^{1/2} < W << L \ln (L/\lambda_F)$



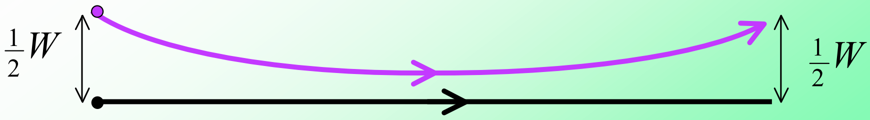
Classical dynamics in phase space

assume: hyperbolic flow at small scales
random flow on large scales



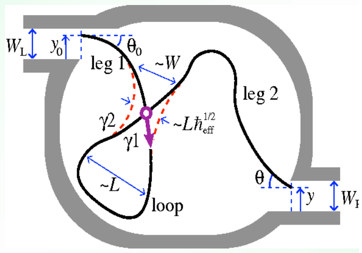


Unfolded hyperbolic dynamics within the band
(relative to black path)



Classical probability of crossing

$\propto$ (phase–space area)$^{-1}$ x $\varepsilon d\varepsilon$ x $dt\; dt_{loop}\; dt_{leg1}$



Uniform escape rate: $\exp[-t/\tau_D]$ for time of path t.

♣ Minimum time for

loop : $t^{loop} = 2\lambda^{-1}\ln[\varepsilon]$ leg : $t^{leg} = \lambda^{-1}\ln[\varepsilon^{-1}(W/L)]$

♣ reduced escape probability within $t^{leg} = \lambda^{-1}\ln[\varepsilon^{-1}(W/L)]$ of crossing

$$\int_{t^{loop}+2t^{leg}}^{\infty} dt\; \exp[-(t-2t^{leg})/\tau_D]\; (t-(t^{loop}+2t^{leg}))^2 \quad \Rightarrow \quad \exp[-2t^{leg}/\tau_D] = \varepsilon^{2/(\lambda\tau_D)}$$

Integral over crossing angles, ε :

(phase–space area)$^{-1}$ x Re $\int \varepsilon d\varepsilon\; \varepsilon^{2/(\lambda\tau_D)} \exp[\, iE_F\, \varepsilon^2/\lambda h]$ => N^{-1}

Is zero without $\varepsilon^{2/(\lambda\tau_D)}$ everything comes from finite length of legs/loop

$g_{wl} \sim$ Drude conductivity x N^{-1}

due to paths > $(t_E^{cl} + t_E^{op})$

Result: $g_{wl} = -1/4 \times \exp[-t_E^{cl}/\tau_D]$