The battle between *interference* and *tunnelling* in chaotic quantum-dots

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GRENOBLE

NEUTRONS FOR SCIENCE

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- ♣ R.W. (submitted to PRB) cond-mat/0612122
- ♣ C. Petitjean, Ph.Jacquod & R.W. (submitted to PRL) cond-mat/0612118

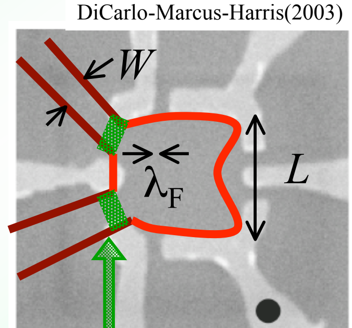
♣ R.W. & Ph. Jacquod, PRL **96**, 206804 (2006)
♣ Ph. Jacquod & R.W., PRB **73**, 195115 (2006)
♣ R.W. & Ph. Jacquod, PRL **94**, 116801 (2005)
See also: Brouwer & co-workers and Haake & co-workers.

CMMP Leicester April 2007

QUANTUM CHAOS : quantum mechanics of a classically chaotic system.

Individual systems: unique
... but average properties: UNIVERSAL

DiCarlo-Marcus-Harris(2003)



tunnel barriers
transparency, ρ

Transport properties through open nanoscale chaotic system:

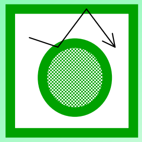
$$\lambda_F << \rho W << L$$

Good conductor
(no Coulomb blockade)

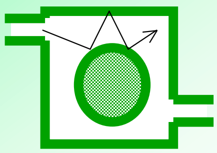
"lots" of chaos

Semiclassical: $\lambda_F << W$

Regimes in closed and open systems



Level-statistics, etc



Conductance, etc

Measure of classicalness: Ehrenfest time/dwell time

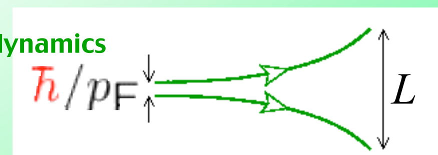
Aleiner-Larkin (1996)

Ehrenfest time, τ_E = time for wavepacket to spread to classical scale

Ehrenfest theorum: spread follows classical dynamics

$$\tau_E = \lambda^{-1} \ln [p_F L/\hbar]$$

Lyapunov exponent



Semiclassics : *ray optics* for the 21st century

Landauer–Buttiker

scattering matrix:

$$S = \begin{pmatrix} r & t^\dagger \\ t & r' \end{pmatrix}$$

CONDUCTANCE : $g = \langle \mathrm{tr}[\mathbf{t}^\dagger \mathbf{t}] \rangle$

SHOT NOISE : intrinsically quantum current noise

Fano factor: $F =$ "noise/current"

$$= g^{-1} \langle \mathrm{tr}[\mathbf{t}^\dagger\mathbf{t} - \mathbf{t}^\dagger\mathbf{t}\,\mathbf{t}^\dagger\mathbf{t}] \rangle$$

Van Vleck/Gutzwiller propagator = *ray optics*

Sum over all classical paths: S_γ = classical action

Semiclassical propagator

⇒ Scattering matrix:

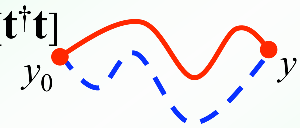
Baranger *et al* (1993)

$$t_{nm} \sim \mathcal{A} \sum_\gamma A_\gamma \exp[i\, S_\gamma / h]$$

Coupling to lead mode wavefunctions

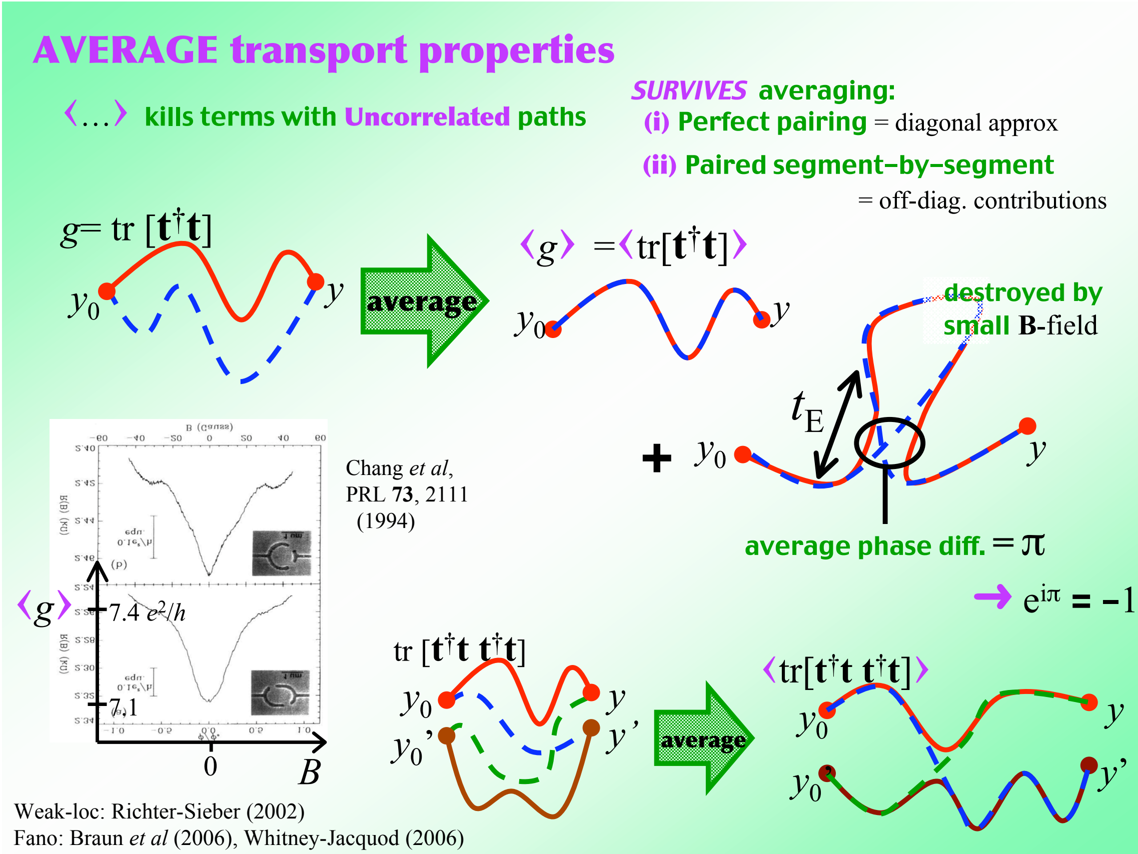
$$\mathcal{A} \sim \int dy_0 \int dy \, \langle n|y\rangle \langle y_0|m\rangle$$

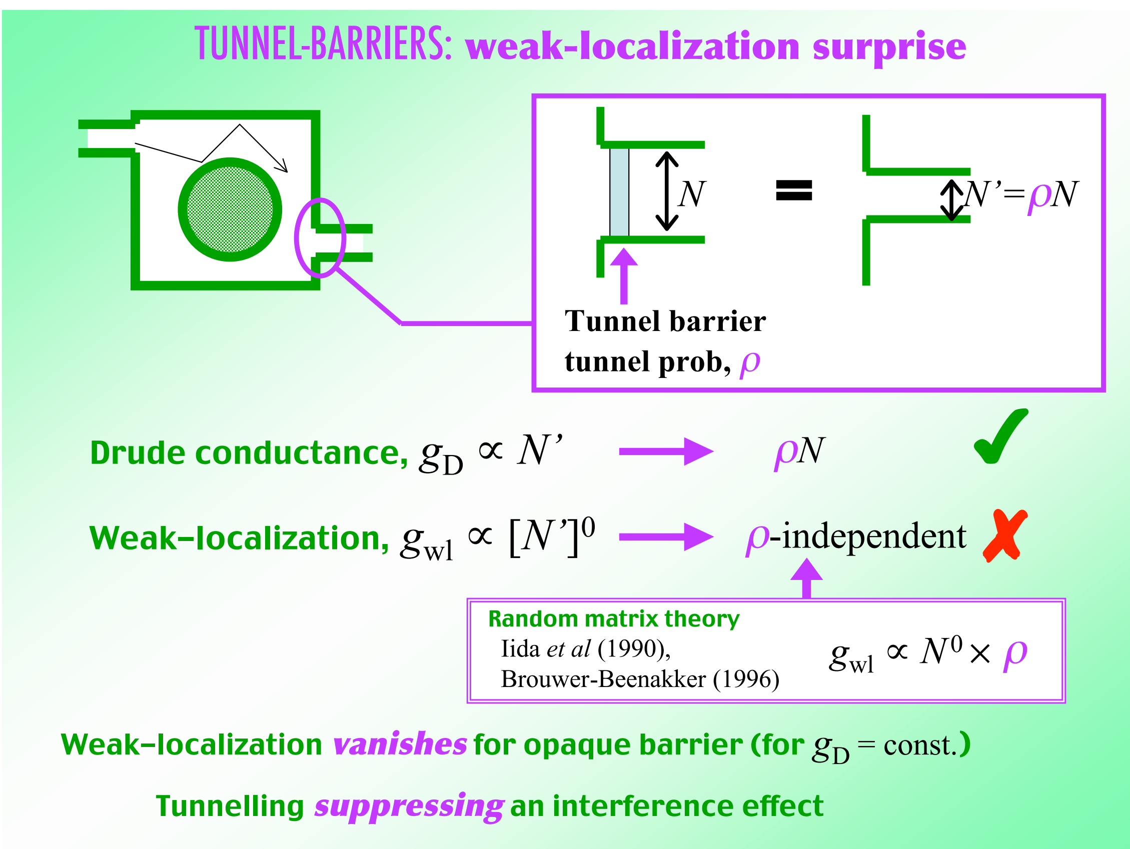
tr $[\mathbf{t}^\dagger\mathbf{t}]$



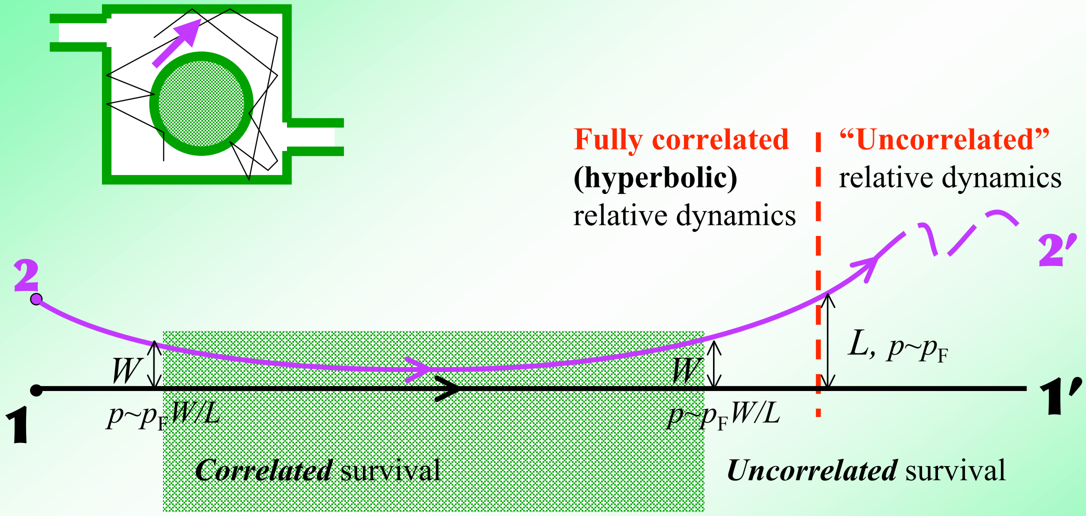
$\langle \ldots \rangle$ supresses terms with Uncorrelated paths

because $\langle \exp[i(S_{\gamma 1} - S_{\gamma 2})/h] \rangle$ killed by oscillation inside $\langle \ldots \rangle$


AVERAGE transport properties
⟨…⟩ kills terms with Uncorrelated paths
SURVIVES averaging:
(i) Perfect pairing = diagonal approx
(ii) Paired segment–by–segment
= off-diag. contributions
g= tr [$\mathbf{t}^\dagger\mathbf{t}$]
y_0
y
average
$\langle g \rangle = \langle$tr[$\mathbf{t}^\dagger\mathbf{t}$]$\rangle$
destroyed by
small B-field
t_E
+ y_0
average phase diff. = π
→ $e^{i\pi}$ = −1
Chang et al,
PRL 73, 2111
(1994)
$\langle g \rangle$
7.4 e^2/h
7.1
0
B
tr [$\mathbf{t}^\dagger\mathbf{t}\ \mathbf{t}^\dagger\mathbf{t}$]
y_0'
y'
⟨tr[$\mathbf{t}^\dagger\mathbf{t}\ \mathbf{t}^\dagger\mathbf{t}$]⟩
Weak-loc: Richter-Sieber (2002)
Fano: Braun et al (2006), Whitney-Jacquod (2006)


TUNNEL-BARRIERS: weak-localization surprise
N
=
$N' = \rho N$
Tunnel barrier
tunnel prob, ρ
Drude conductance, $g_D \propto N'$ → ρN ✔
Weak–localization, $g_{wl} \propto [N']^0$ → ρ-independent ✘
Random matrix theory
Iida et al (1990),
Brouwer-Beenakker (1996)
$g_{wl} \propto N^0 \times \rho$
Weak–localization vanishes for opaque barrier (for g_D = const.)
Tunnelling suppressing an interference effect

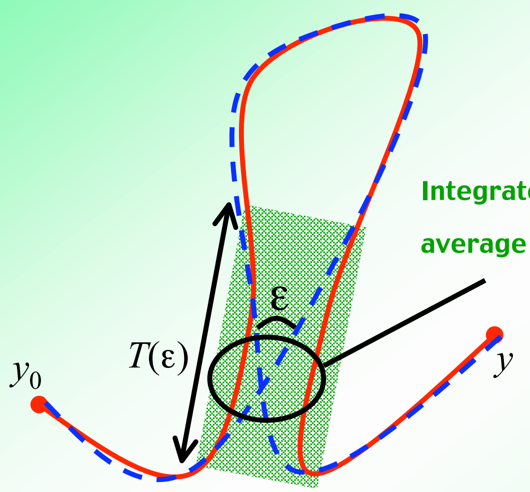
Modelling *"generic"* classical chaos



Also classical paths 1 → 2′ and 2 → 1′

no tunnel–barriers: Sieber-Richter (2001,02), Rahav-Brouwer (2005), Jacquod-Whitney (2006)
With tunnel–barriers: Petitjean-Jacquod-Whitney (2007), Whitney (2007).

Weak-localization result with tunnel-barriers



Integrate over ε **:** dominated by $\varepsilon \approx (\lambda_F/L)^{1/2}$

average phase diff. $= \pi$ ➔ $e^{i\pi} = -1$

$T(\varepsilon)$ ➔ **Ehrenfest time**

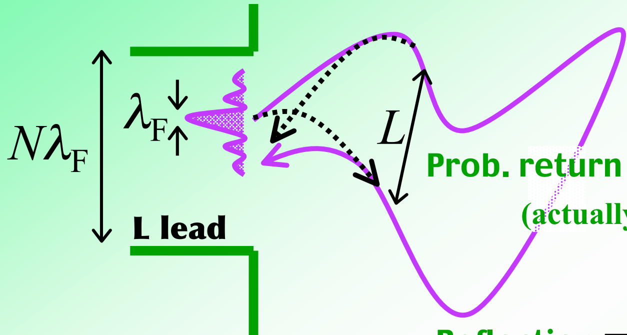
weak–loc = Drude conductance x $\alpha \dfrac{\tau_{D2}\ \exp[-\tau_E/\tau_{D2}]}{\text{phase-space area}}$

≈ result *without* tunnel–barriers
neglecting $\mathcal{O}[1]$-factors like α

✘ *...so where did we go wrong??*

Coherent back-scattering peak

Jacquod-Whitney (2006)
Rahav-Brouwer (2006)



Prob. return approx ***doubled*** within λ_F
(actually in phase-space not real space)

Reflection $= N/2 \times [1 + 1/N - 1/(2N)]$

Weak-localization ≡ Reduced prob for all other paths, half of which reflect

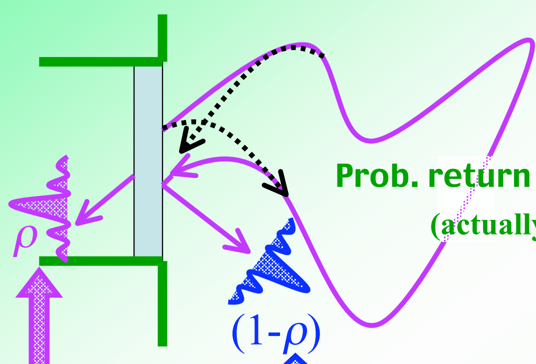
conductance = transmission $= N/2 \times [1 - 1/(2N)]$

Weak-localization $\sim N^0$

> **However minimum time for loop means**
> 1 & 1 are actually $\exp[-\tau_E/\tau_D]$ $\xrightarrow{\text{classical limit}}$ 0

Coherent back-scattering with tunnel-barrier

Whitney (2007)



Prob. return approx ***doubled*** within λ_F
(actually in phase-space not real space)

Successful coherent-backscatter

Failed coherent-backscatter

Reflection $= N/2 \times [1 + (1/N)(\rho + (1-\rho)/2) - 1/(2N)]$

conductance $= N/2 \times [1 + (1/2N)(1-\rho) - 1/(2N)] = N/2 - 1/4 \times \rho$

✔ *Correct* weak-localization result $\sim N^0 \times \rho$

Weak-localization *suppressed* by tunnelling.

In both RMT regime and in cross-over to classical regime.

(even without tunnelling, weak-loc=0 *in* classical regime)

Result known in RMT regime since 1990 - Iida *et al* (1990), Brouwer-Beenakker (1996)

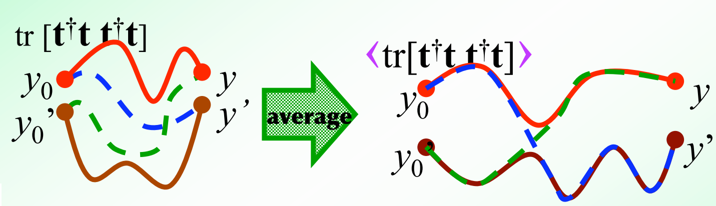
...but no physical picture of *why*.

Simple physical origin:

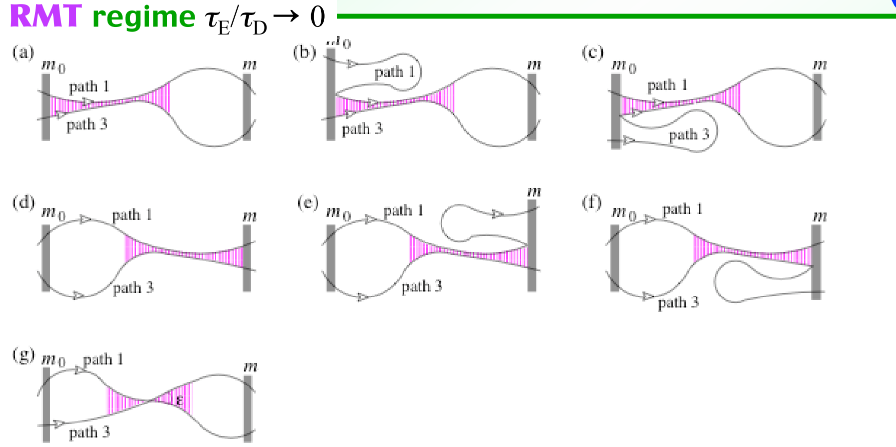
Tunnel-barriers *smear* coherent back-scattering peak between reflection and transmission

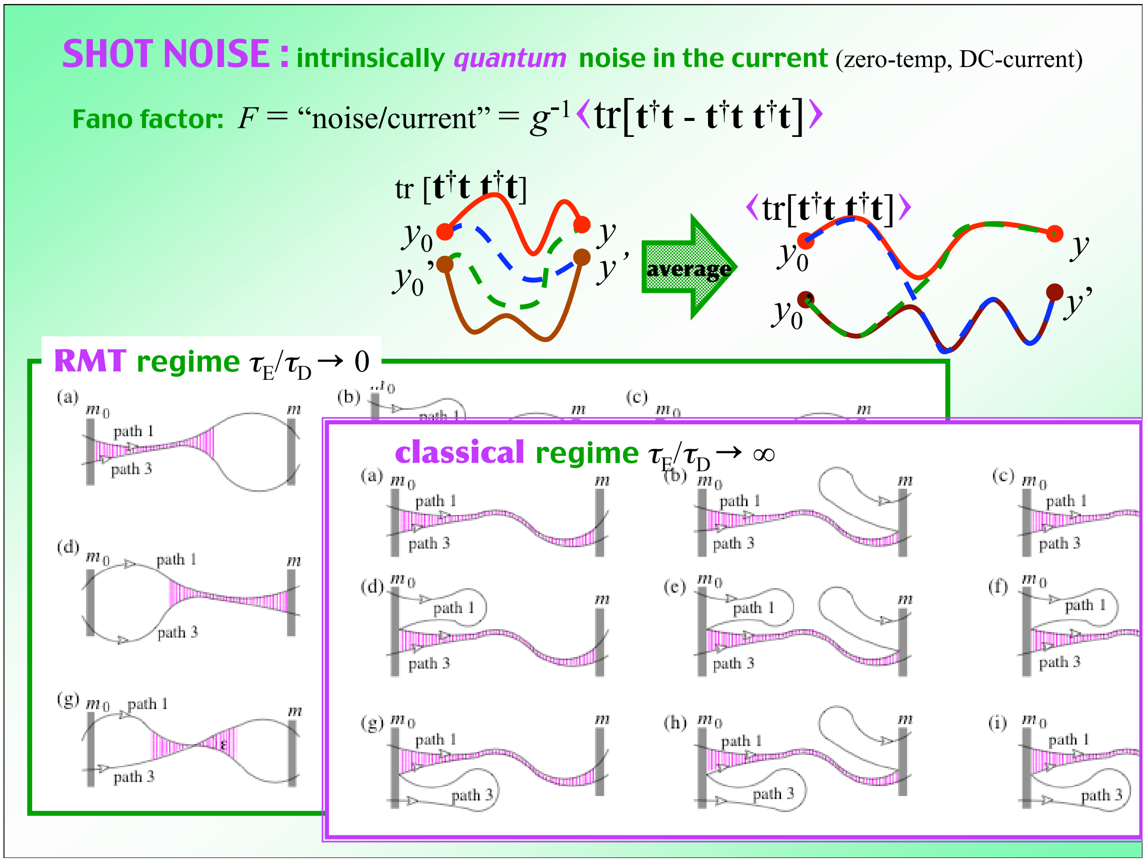
SHOT NOISE : intrinsically *quantum* noise in the current (zero-temp, DC-current)

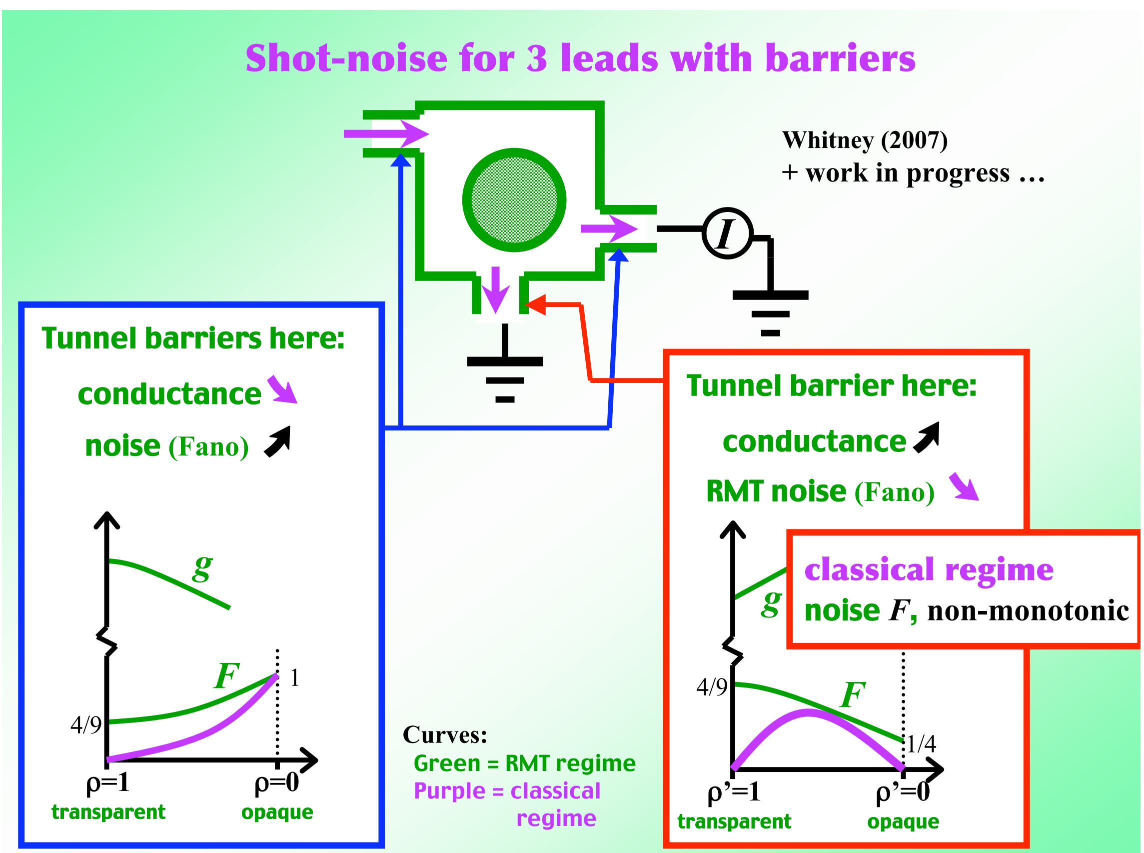
Fano factor: F = "noise/current" = $g^{-1}\langle \mathrm{tr}[\mathbf{t}^\dagger\mathbf{t} - \mathbf{t}^\dagger\mathbf{t}\,\mathbf{t}^\dagger\mathbf{t}]\rangle$



RMT regime $\tau_E/\tau_D \rightarrow 0$




SHOT NOISE : intrinsically quantum noise in the current (zero-temp, DC-current)
Fano factor: F = "noise/current" = g^-1 ⟨tr[t†t - t†t t†t]⟩
tr [t†t t†t]
y0
y0'
y
y'
average
⟨tr[t†t t†t]⟩
y0
y0'
y
y'
RMT regime τE/τD → 0
(a) m0 path 1 path 3 m
(b) path 1 m
(c) m0 m
(d) m0 path 1 path 3 m
(g) m0 path 1 path 3 m
classical regime τE/τD → ∞
(a) m0 path 1 path 3 m
(b) m0 path 1 path 3 m
(c) m0 path 1 path 3
(d) m0 path 1 path 3 m
(e) m0 path 1 path 3 m
(f) m0 path 1 path 3
(g) m0 path 1 path 3 m
(h) m0 path 1 path 3 m
(i) m0 path 1 path 3


Shot-noise for 3 leads with barriers
Whitney (2007)
+ work in progress …
I
Tunnel barriers here:
conductance ↘
noise (Fano) ↗
g
F
4/9
1
ρ=1
transparent
ρ=0
opaque
Curves:
Green = RMT regime
Purple = classical regime
Tunnel barrier here:
conductance ↗
RMT noise (Fano) ↘
classical regime
noise F, non-monotonic
g
4/9
F
1/4
ρ'=1
transparent
ρ'=0
opaque

Conclusions

Weak-localization *suppressed* by tunnelling.

In both RMT regime and in cross-over to classical regime.

(even without tunnelling, weak-loc=0 *in* classical regime)

Simple physical origin:

Tunnel-barriers *smear* coherent back-scattering peak between reflection and transmission

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Noise (with tunnel barriers) **in classical regime is very different from RMT regime**

Barrier on 3rd lead → ***reduce* noise in RMT regime**

→ ***reduce* or *enhance* noise in classical regime**