Presentation for
Weizmann Node

Weizmann Institute of Science
מכון ויצמן למדע

Berry phase in a non-isolated system and geometric dephasing

Rob Whitney[1,]

Yu. Makhlin[2], A. Shnirman[3], and Y. Gefen[4]

Preprint: cond-mat/0405267

see also Proc. NATO-ASI cond-mat/0401376
& *Phys. Rev. Lett.* 90, 190402 (2003)

Current addresses : 1. Univ. Genéve. 2. Landau Inst. 3. Univ. Karlsruhe. 4. Weizmann Inst.

Overview

- Berry phase in **spintronics** : solid state realisation of Berry phase
- Is Berry phase observable for **non-isolated** system?

  Naïve arguments say **NO**
  Experiments say **YES**
  } Resolving the **contradiction**

Overview

- Berry phase in **spintronics** : solid state realisation of Berry phase
- Is Berry phase observable for **non-isolated** system?

  Naïve arguments say **NO**
  Experiments say **YES**
  } Resolving the **contradiction**

- Berry phase **modified** by environment :
  - ♣ remains **geometric** (but longer monopole)
  - ♣ becomes **complex** ⇒ geometric dephasing
- **Geometric dephasing** : can increase or **decrease** dephasing
  - ♣ Well-defined (gauge-**independent**) for **open** paths

Berry phase for (isolated) spin-half

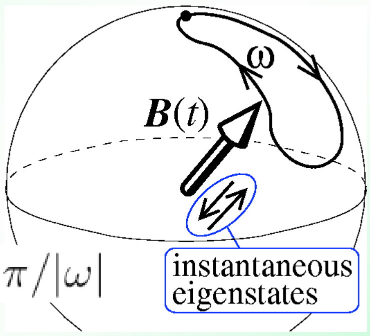
Berry (1984)

Slowly varying B-field : $|\omega| \ll |\boldsymbol{B}|$

$$P(\,|\uparrow\rangle \to |\downarrow\rangle\,) \sim (Bt)^{-1} \ll 1$$

i.e. Adiabatic evolution

$t \sim 2\pi/|\omega|$

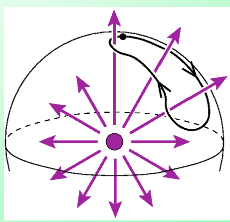


$$\Phi = \Phi_{\text{Dynamic}} + \Phi_{\text{Berry}} + \mathcal{O}[(Bt)^{-1}]$$

$$\Phi_{\text{Dynamic}} = \tfrac{1}{2}|\boldsymbol{B}|t$$

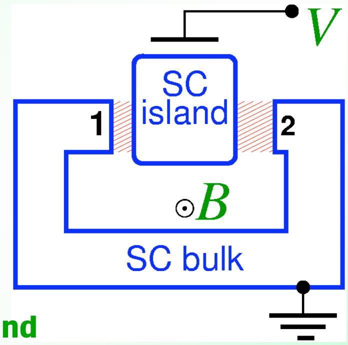
$$\Phi_{\text{Berry}} = \tfrac{1}{2}\,\text{(enclosed solid angle)}$$

$$= \tfrac{1}{2}\,\text{(flux of monopole thru loop)}$$



Berry phase in spintronics?

Potential solid-state realisation



Berry phase in Superconducting nanocircuit(qubit) Falci *et al* (2000)

$$\hat{\mathcal{H}} = E_{\rm C}\left(\hat{n} - n_V\right)^2 - E_{\rm J}(B)\cos\left[\hat{\theta} - \alpha_B\right]$$

Consider only lowest 2 charge-states of island

$$|\uparrow\rangle \equiv |n\rangle \quad \& \quad |\downarrow\rangle \equiv |n+1\rangle$$

Reduced Hamiltonian: $$\hat{\mathcal{H}} = \begin{pmatrix} E_{\rm J}\cos(\alpha_B) \\ E_{\rm J}\sin(\alpha_B) \\ E_{\rm C}(1-n_V) \end{pmatrix} \cdot \hat{\sigma}$$

Environment? ♣ charge fluctuations couple via σ_z
♣ current fluctuations couple via σ_x, σ_y

Spin-half coupled to environment

- **Spin coupled to quantum environment :**

$$\mathcal{H} = \mathcal{H}_{\text{system}} + \mathcal{H}_{\text{interaction}} + \mathcal{H}_{\text{environ}}$$

$$\mathcal{H}_{\text{system}} = -\tfrac{1}{2}\boldsymbol{B}(t)\,.\,\boldsymbol{\sigma} \qquad \mathcal{H}_{\text{interaction}} = -\tfrac{1}{2}\sum_n C_n\,(a_n^\dagger + a_n)\,\sigma_z$$

where $a_n^\dagger, a_n$ create/annihilate nth environment mode

For example: **spin-boson model** Leggett *et al* (1986–87)

Environ. = Oscillators with smooth distrib. of freq.

Spin-half coupled to environment

- Spin coupled to quantum environment :

$\mathcal{H} = \mathcal{H}_{\text{system}} + \mathcal{H}_{\text{interaction}} + \mathcal{H}_{\text{environ}}$

$\mathcal{H}_{\text{system}} = -\frac{1}{2}\boldsymbol{B}(t)\,.\,\boldsymbol{\sigma}$ $\qquad$ $\mathcal{H}_{\text{interaction}} = -\frac{1}{2}\sum_n C_n\,(a_n^\dagger + a_n)\,\sigma_z$

where $a_n^\dagger$, a_n create/annihilate nth environment mode

For example: spin-boson model Leggett *et al* (1986-87)

Environ. = Oscillators with smooth distrib. of freq.

- Spin coupled to noisy classical B-field, $\boldsymbol{K}(t)$:

$\mathcal{H} = -\frac{1}{2}\,[\boldsymbol{B}(t) + \boldsymbol{K}(t)].\boldsymbol{\sigma}$ where $\langle \boldsymbol{K}(\tau)\,\boldsymbol{K}(0)\rangle = C^2 \times f(\tau)$

Equiv. to quantum problem. Whitney-Makhlin-Shnirman-Gefen (2004)

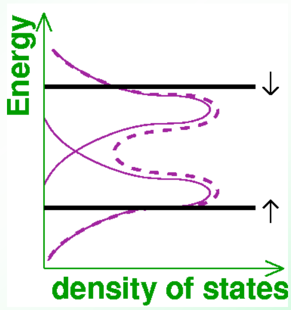
Berry phase and with dephasing?

Environment induces level-broadening

$\Rightarrow$ No Gap

$P(\,|\!\uparrow\rangle \rightarrow |\!\downarrow\rangle\,) \rightarrow 1$ as $t \rightarrow \infty$

No Adiabaticity $\Rightarrow$ No Berry phase



Whitney-Gefen, *PRL* (2003)

BUT : All real expts are non-isolated, yet Berry phase is observed

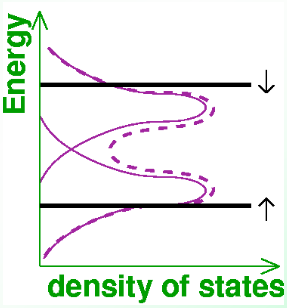
Berry phase and with dephasing?

Environment induces level-broadening

$\Rightarrow$ No Gap

$$P(\,|\uparrow\rangle \to |\downarrow\rangle\,) \to 1 \quad \text{as} \quad t \to \infty$$

No Adiabaticity $\Rightarrow$ No Berry phase



Whitney–Gefen, *PRL* (2003)

BUT : All real expts are non-isolated, yet Berry phase is observed

Berry phase is observable whenever

adiabatic time << dephasing time

$$\hbar/E_{\rm gap} \ll T_2$$

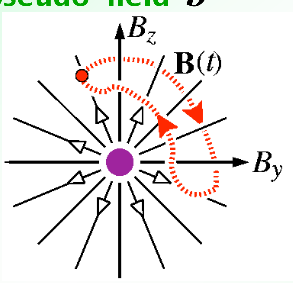
i.e. require small matrix elements for transitions not a true gap

Env.-induced modification of the Berry phase

get phase as $\oint$ along path of $\boldsymbol{B}(t)$ $\Rightarrow$ use Stokes' theorum

$\Rightarrow$ surface int. $\Phi_{\rm Berry} = \int {\rm d}\boldsymbol{S}\cdot\boldsymbol{b}$

monopole pseudo-field $\boldsymbol{b}$



Amplitude of monopole = 1/2

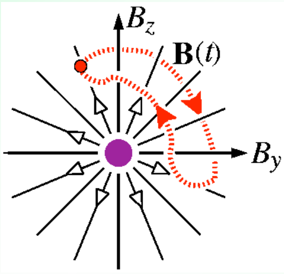
Env.-induced modification of the Berry phase

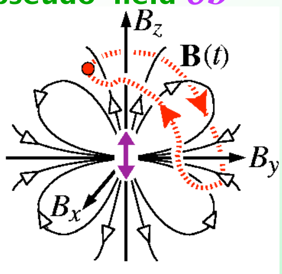
get phase as $\oint$ along path of $\boldsymbol{B}(t)$ $\Rightarrow$ use Stokes' theorum

$\Rightarrow$ surface int. $\Phi_{\mathbf{Berry}} = \int \mathrm{d}\boldsymbol{S}\cdot(\boldsymbol{b}+\delta\boldsymbol{b})$

monopole pseudo-field $\boldsymbol{b}$

"quadrupole" pseudo-field δb





Angular $=$ $Y_{20}(\theta,\varphi)$
Radial $\neq B^{-4}$

(non-zero curl)

Amplitude of monopole $= 1/2$
Amplitude of "quadrupole" $= C^2 \times$ complex function (env. spectrum)

Berry phase as derivative of gap

For isolate spin : $\Phi_{\mathbf{Berry}} = \int_0^t \mathrm{d}t'\omega_z \frac{\mathrm{d}|\boldsymbol{B}|}{\mathrm{d}B_z} = \oint \mathrm{d}\varphi \frac{\mathrm{d}}{\mathrm{d}B_z} B$

For NON-isolate spin: $B \rightarrow [B + \delta B + \mathrm{i}\Gamma_2]$

$\delta B =$ Lamb shift of energy level
$\Gamma_2 =$ dephasing rate

Berry phase as derivative of gap

For isolate spin : $\Phi_{\mathrm{Berry}} = \int_0^t \mathrm{d}t' \omega_z \frac{\mathrm{d}|\boldsymbol{B}|}{\mathrm{d}B_z} = \oint \mathrm{d}\varphi \, \frac{\mathrm{d}}{\mathrm{d}B_z} B$

For NON-isolate spin: $B \rightarrow [B + \delta B + \mathrm{i}\Gamma_2]$

$\delta B =$ **Lamb shift of energy level**

$\Gamma_2 =$ **dephasing rate**

Pretty result : $\Phi_{\mathrm{Berry}} = \oint \mathrm{d}\varphi \, \frac{\mathrm{d}}{\mathrm{d}B_z} [B + \delta B + \mathrm{i}\,\Gamma_2]$

Revisit this later

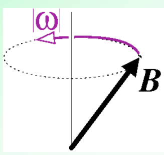
- ♣ **Berry phase is complex if spin dephases**
- ♣ **Real (“phase”) part is modified if there is Lamb shift**

Geometric dephasing

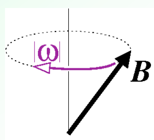
Imaginary part of Berry phase ⇒ dephasing

$$\mathrm{Im}[\Phi_{\mathrm{Berry}}] = \int_0^t \mathrm{d}t' \, \omega_z \frac{\mathrm{d}\Gamma_2}{\mathrm{d}B_z}$$

Can be either sign; depends of direction of winding



⟹ $\omega_z > 0$: **geometric dephasing *positive***
⇒ **increase total dephasing**



⟹ $\omega_z < 0$: **geometric dephasing *negative***
⇒ **REDUCES total dephasing**

...but it is only a small modification of total dephasing

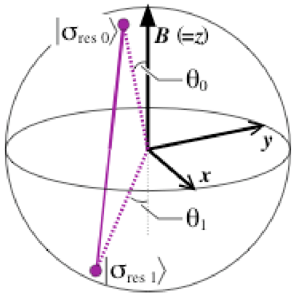
How do we understand these results?

To find out
tune in after the advert break …

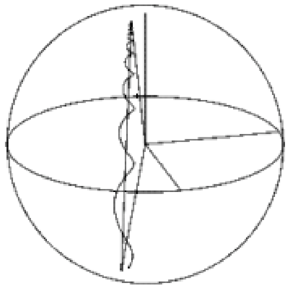
Poster this evening

Dynamics and **non-orthogonal** resonances
of a **non-isolated spin**

spin-resonances



spin-dynamics



+ all technical details of this talk

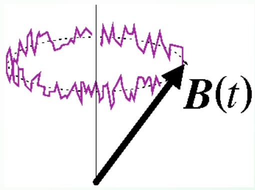
How do we understand these results?

♣ **Toy problem : Noisy classical field**

(Gaussian white-noise)

Whitney-Gefen, *Proc. Moriond* (2001)

Whitney-Makhlin-Shnirman-Gefen, *Proc. NATO-ARW* (2004)



♣ **Fully quantum problem :**

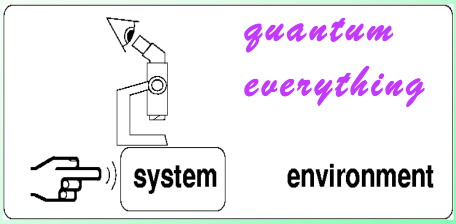
coupling many environ. modes, trace them out

Use rotating frame trick

Whitney-Gefen, *PRL* (2003)

Whitney-Makhlin-Shnirman-Gefen,

to be published (2004)



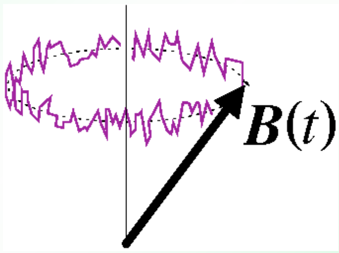
Noisy classical field

Toy problem : Gaussian white-noise

$$\mathcal{H} = -\tfrac{1}{2}[\boldsymbol{B}(t) + \boldsymbol{K}(t)].\boldsymbol{\sigma}$$

where $\langle K(\tau)K(0)\rangle_K = C^2\delta(\tau)$

Whitney-Gefen (2001)



Adiabatic evolution during one-time step

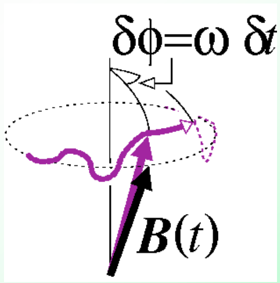
Noisy classical field

Toy problem : Gaussian white-noise

$$\mathcal{H} = -\tfrac{1}{2}[\boldsymbol{B}(t) + \boldsymbol{K}(t)].\sigma$$

where $\langle K(\tau)K(0)\rangle_K = C^2\delta(\tau)$

Whitney–Gefen (2001)



Adiabatic evolution during one-time step

$$\left\langle \exp\left[\mathrm{i}|\mathbf{B}+\mathbf{K}|\delta t + \mathrm{i}\delta\phi\, \cos\theta_{(\mathbf{B}+\mathbf{K})} \right] \right\rangle_K$$

$$\langle ... \rangle_K = \int \mathrm{d}K (...) \exp[-K^2 \delta t / C^2]$$

$\langle \cos\theta_{(\mathbf{B}+\mathbf{K})} \rangle$ ⇒ Modification of real (phase) geometric term

cross-terms in completed squ.

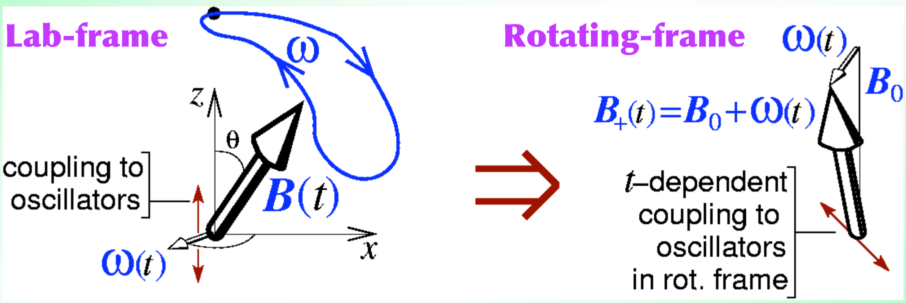
⇒ Imaginary part of geometric term

⇒ geometric dephasing

Phase in rotating frame ⇒ Berry phase

⇒ **Rotating frame — rotates with B–field :**

Hamiltonian ≈ time–independent



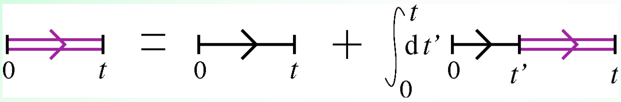
Pseudo–forces/fields ⇒ Berry phase

Solve time–independent problem in rotating frame

⇒ **solution of time–dependent problem in Lab frame**

Technicalities of Master eqn

Real time Dyson eqn :



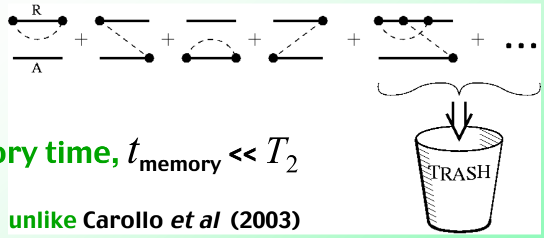
⇒ exact **Master equation** for spin's **density matrix**

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\mathrm{i}[\mathcal{H}_{\mathbf{sys}}, \rho(t)] + \int_0^t \mathrm{d}\tau\, \boldsymbol{\Sigma}(\tau)\rho(t-\tau)$$

Bloch-Redfield (1957), Schoeller-Schon(1994)

⇒ systematic **weak-coupling** and **Markov** approximations

self-energy; $\Sigma(\tau) =$



+ Assume environ. memory time, $t_{\text{memory}} \ll T_2$

...but not $t_{\text{memory}} \ll 1/B$, unlike Carollo *et al* (2003)

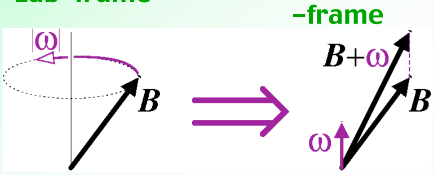
Understanding diff. eqn. for Berry phase

Revisiting : Berry phase as **differential of gap**;

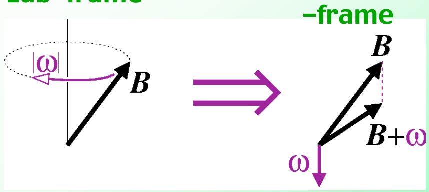
$$\Phi_{\mathbf{Berry}} = \oint \mathrm{d}\varphi \, \frac{\mathrm{d}}{\mathrm{d}B_z}[B + \delta B + \mathrm{i}\,\Gamma_2]$$

Equation is easily understood by going to **rotating frame**

Lab-frame Rotating -frame



Lab-frame Rotating -frame

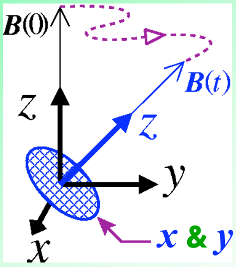


Lamb shift, δB, and dephasing rate, Γ_2, both **scale with gap** $\propto |B+\omega|$

⇒ Taylor expand in ω

⇒ both have ω-terms ⇒ **Geometric terms**

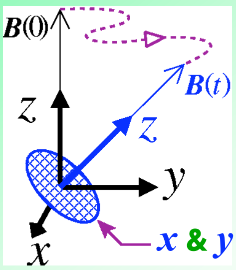
Gauge-independence for open paths?



Ambiguity in choice of x & y axes
$\Rightarrow$ gauge-dependence
of Berry phase for open paths

x & y axes somewhere in this plane

Gauge-independence for open paths?



Ambiguity in choice of x & y axes
$\Rightarrow$ gauge-dependence
of Berry phase for open paths

x & y axes somewhere in this plane

Dephasing affects magnitude of off-diag. element of density matrix

Off–diag. matrix element $= \langle\sigma_x\rangle \pm \mathrm{i}\langle\sigma_y\rangle$

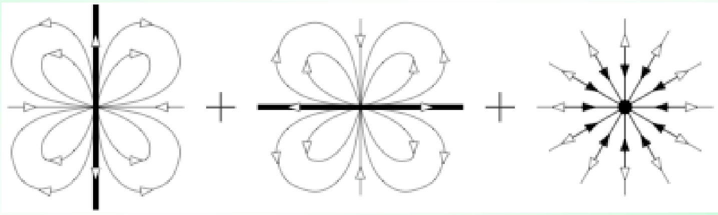
Magnitude $= \sqrt{\langle\sigma_x\rangle^2 + \langle\sigma_y\rangle^2}$

Independent of choice of x & y axes $\Rightarrow$ gauge-independent

Anisotropy is required

Consider **isotropic** coupling to environment

Isotropic ≡ **z-axis** coupling + **y-axis** coupling + **x-axis** coupling



All three couplings equal
➡ all three **"quadrupoles"** have **equal** strength

"Quadrupoles" sum to **ZERO**
➡ Berry phase **unmodified** by environment
➡ No geometric dephasing

Summary

- Berry phase observable for **weak**-dissipation; $BT_2 >> 1$
  i.e. small **matrix elements** for spin-flip

- For **anisotropic environment :**
  Berry phase **modified** by environment
  ➡ **geometric** = quadrupole-like
  ➡ **complex** funct. of env. spectrum

- **Geometric dephasing** : can increase or **decrease** dephasing
  ♣ Well-defined (gauge-**independent**) for **open** paths