Institut Laue Langevin, Grenoble, France.

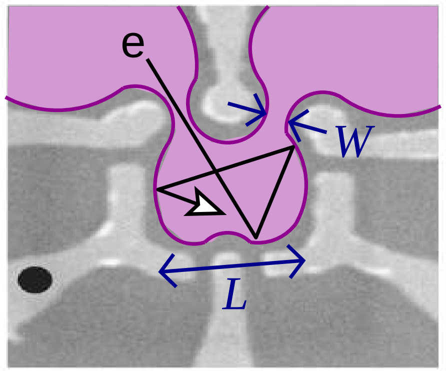
Mirror symmetries in quantum chaos

Robert S. Whitney

Collaborators:

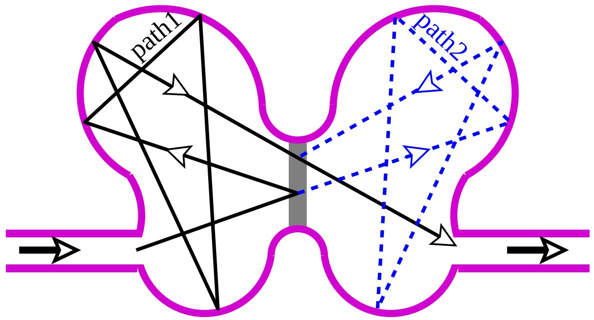
Paolo Marconcini, Massimo Macucci (Università di Pisa)

Henning Schomerus, Marten Kopp (University of Lancaster)



Quantum dot

Marcus Group, Harvard (2003)

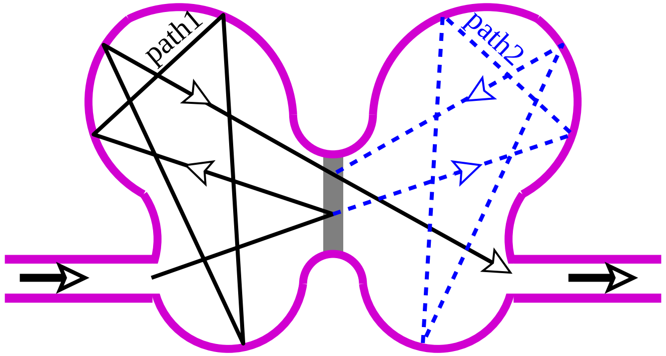


GDR Physique Quantique Mésoscopique - Aussois, Dec 2008

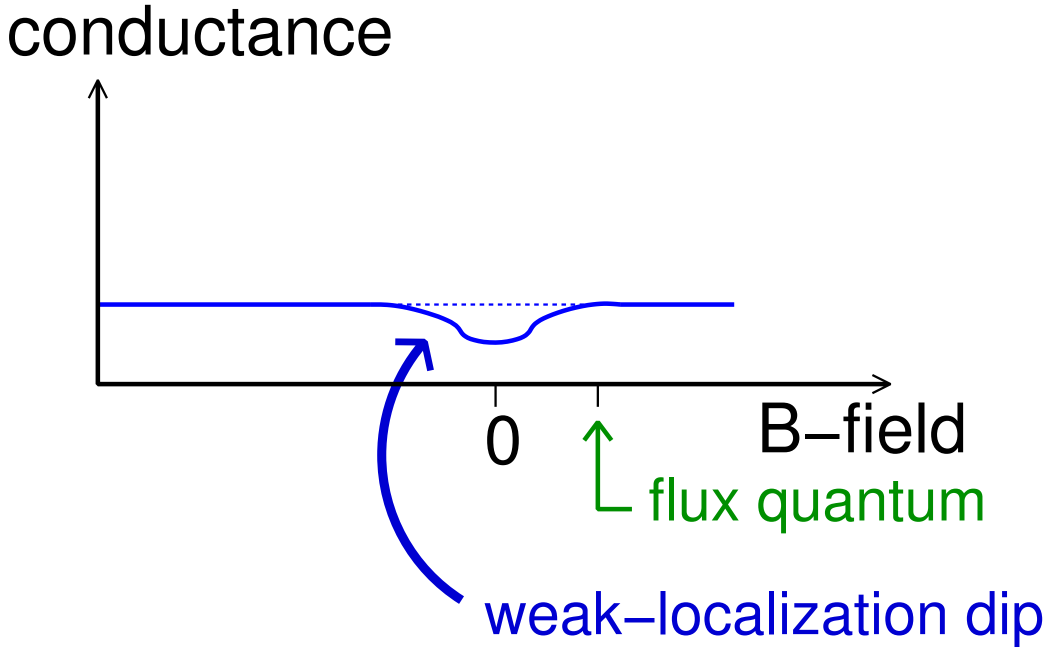
Mirror-symmetric dots - expectations and surprises

Expectations: Baranger-Mello (1996) – symmetric dot without barrier

Double-dot with barrier in middle



lead & dot size $\gg \lambda_F$



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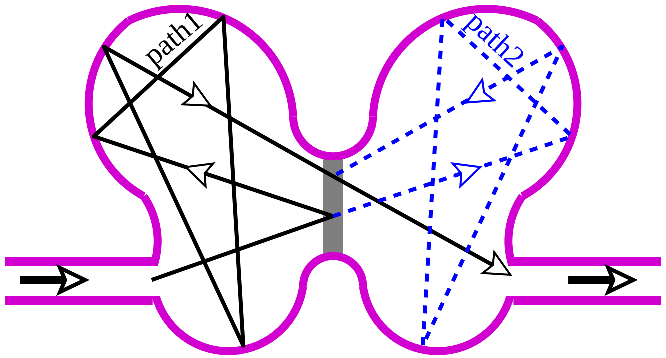

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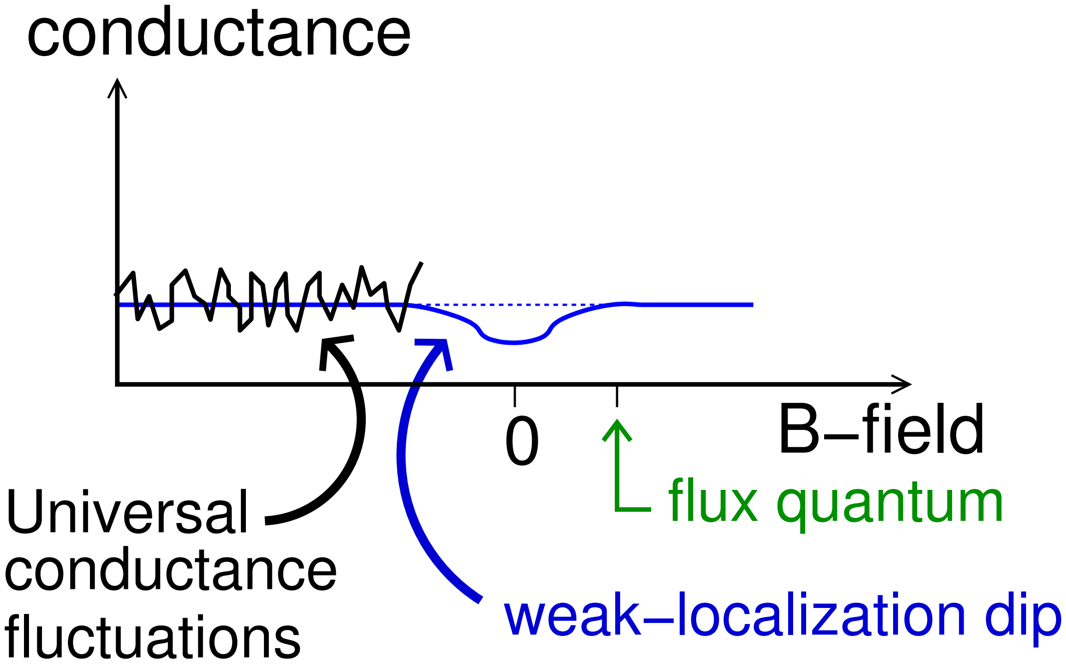

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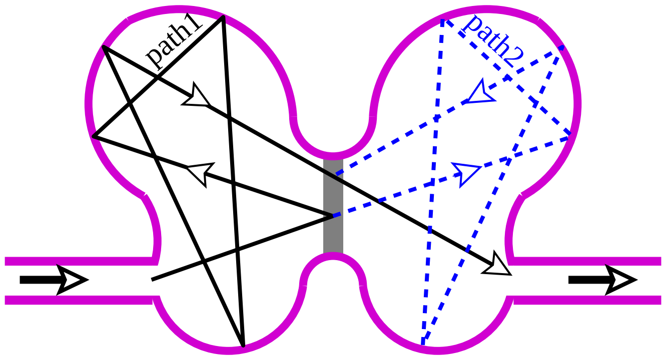

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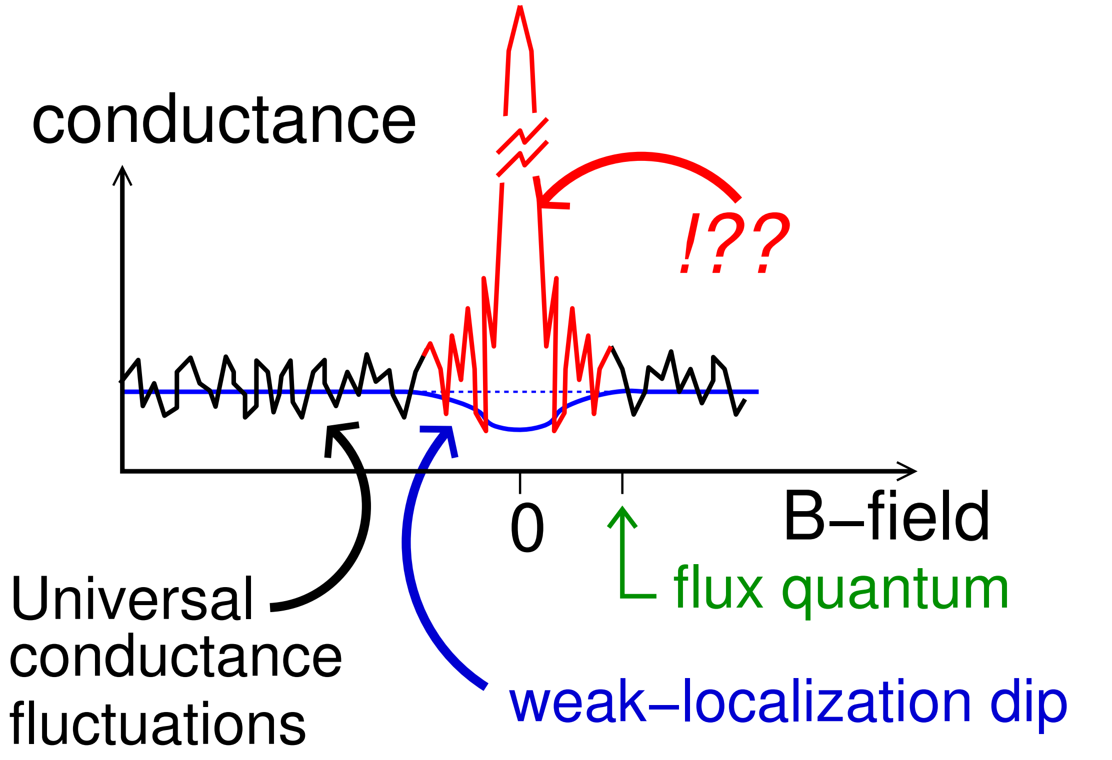

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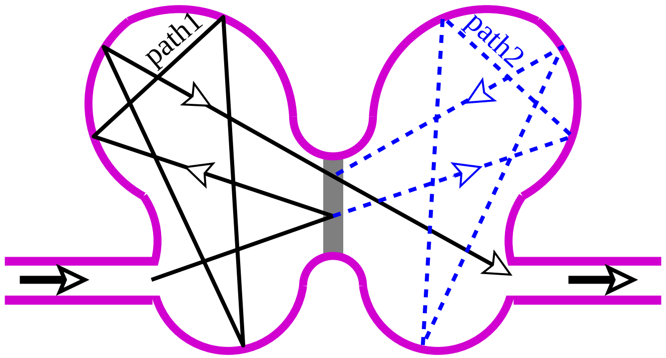

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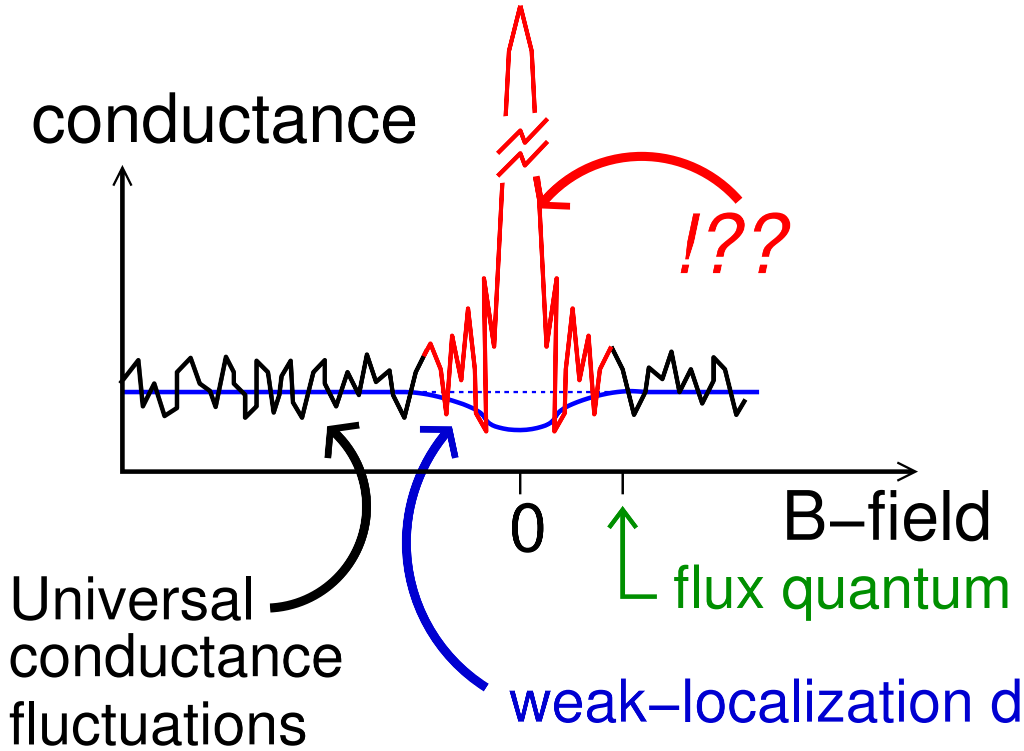


Outline of talk:

- Interference between mirror-symetric paths makes barrier almost "invisible" $\Rightarrow$ huge peak
- symmetry breaking (B-field, disorder, etc)
- *Guessing* experimental numbers

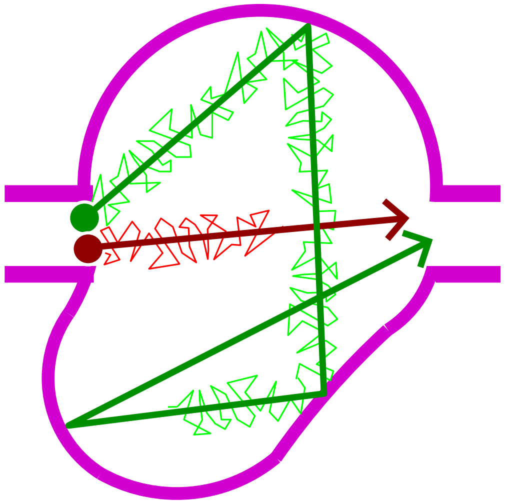
Semiclassics : ray optics in 21st century

wavelength $\ll$ other scales

saddle-point of Feynman path integral

$\Longrightarrow$ classical paths

plus interference $\propto \cos\left[(S_{\gamma 1} - S_{\gamma 2})/\hbar\right]$



Classical paths: *lots* of chaos

$\Longrightarrow$ many bounces before escape

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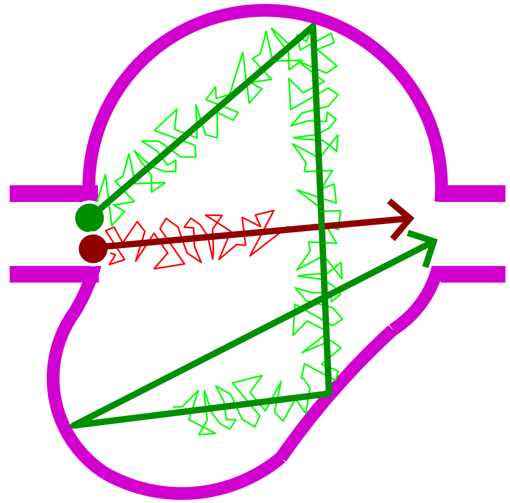

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Classical paths: *lots* of chaos

$\Longrightarrow$ many bounces before escape

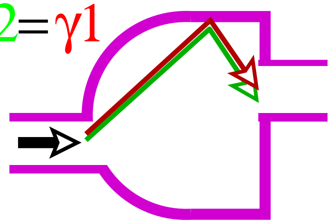
Transmission probability

2 Feynman integrals $\longrightarrow$ 2 paths

$$\Longrightarrow \sum_{\gamma 1, \gamma 2 \in \text{classical paths}} \sqrt{P_{\gamma 1} P_{\gamma 2}}\; \mathrm{e}^{\mathrm{i}(S_{\gamma 1} - S_{\gamma 2})/\hbar}$$

CLASSICAL

$\gamma 2 = \gamma 1$

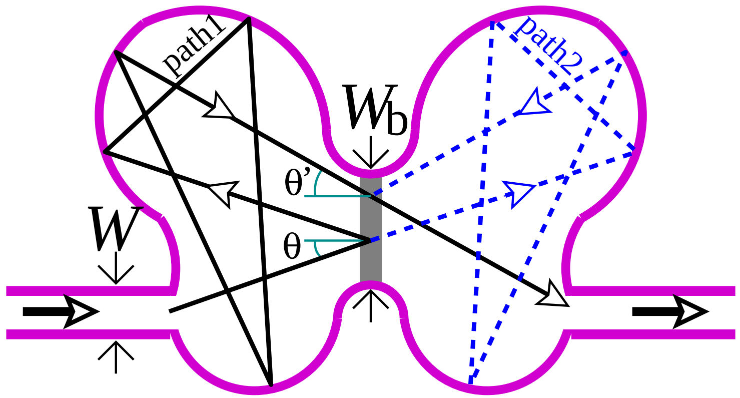


INTERFERENCE

$\gamma 2 \neq \gamma 1$

... *but* with symmetry??

"Butterfly" double-dot



Handwaving:

take only 2 paths

$$\text{Quantum probability} = \left| rt e^{iS_1/\hbar} + tr e^{iS_2/\hbar} \right|^2$$

$$\longrightarrow \begin{cases} 2|rt|^2 & \text{asymmetric } (S_2 \neq S_1) \\ 4|rt|^2 & \text{symmetric } (S_2 = S_1) \end{cases}$$

Symmetric $= 2 \times$ asymmetric $\Leftarrow$ if θ-independent

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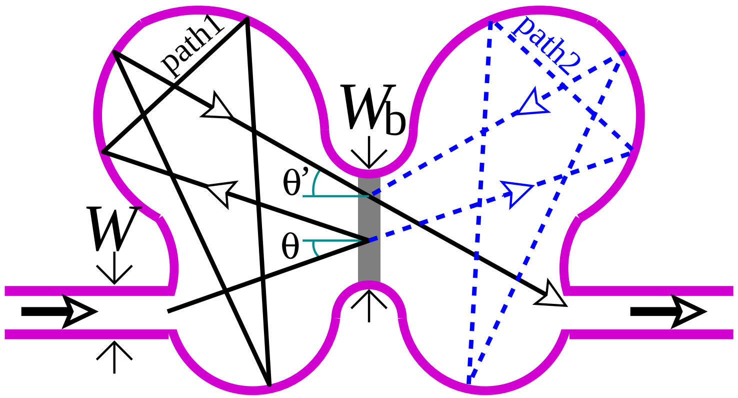

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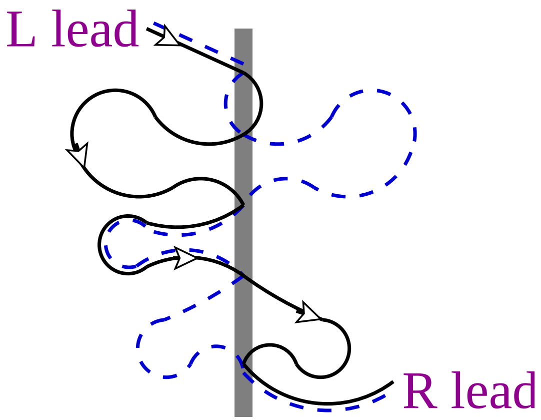
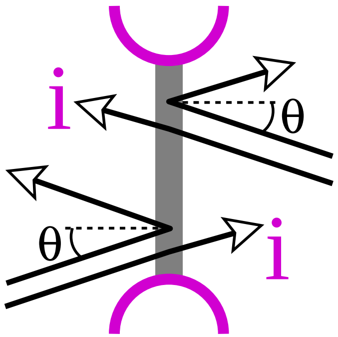


Path hits barrier $(n+1)$ times $\Rightarrow 2^n$ partners

Symmetric $\simeq 2^n \times$ asymmetric †

† except we forgot phases at barrier

Double-dot: beyond handwaving



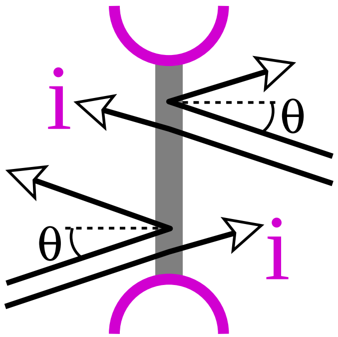
... keeping phases at barrier

$$\mathbf{S}_\theta = \mathrm{e}^{\mathrm{i}\phi} \begin{pmatrix} |r_\theta| & \mathrm{e}^{\mathrm{i}\pi/2}\,|t_\theta| \\ \mathrm{e}^{\mathrm{i}\pi/2}\,|t_\theta| & |r_\theta| \end{pmatrix}$$

$\Rightarrow$ *Destructive interference* between 2 paths

if one tunnels $2(2j-1)$ times more than the other

Double-dot: beyond handwaving



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⇒ *Destructive interference* between 2 paths

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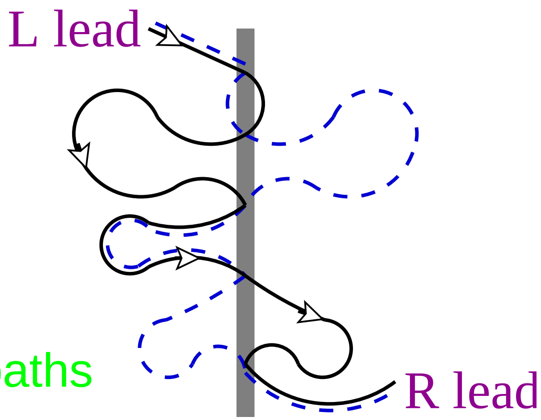


Quantum prob. $\propto (1-P)\sum_{n=0}^{\infty} P^n \left[\boldsymbol{\mathcal{S}}_{\rm b}^n\right]_{41}$

– P = prob. to hit barrier

– $\boldsymbol{\mathcal{S}}_{\rm b}$ = double-scattering matrix (4×4) acts BOTH paths

— acts on (LL,LR,RL,RR)

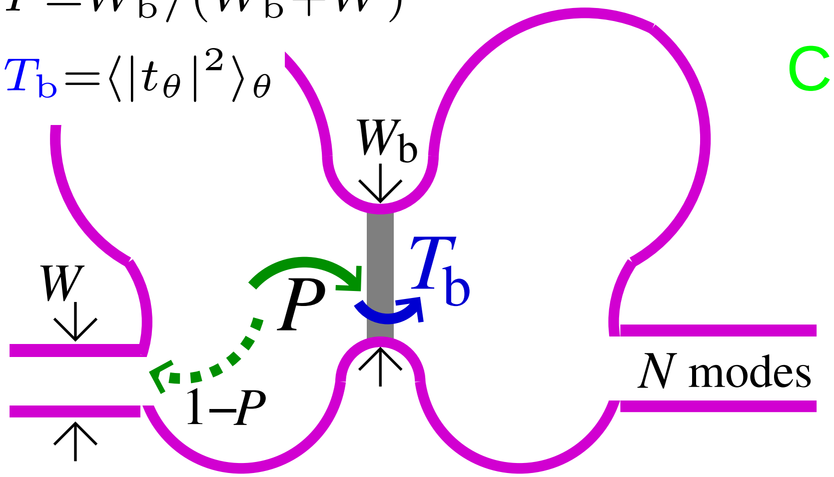
• put asymmetry in $\boldsymbol{\mathcal{S}}_{\rm b}$: put $\exp[i\delta S/\hbar]$ in non-corner terms

⇒ diagonalize $\boldsymbol{\mathcal{S}}_{\rm b}$ and sum geometric series in n

Size of conductance-peak

$P = W_b/(W_b + W)$

$T_b = \langle |t_\theta|^2 \rangle_\theta$



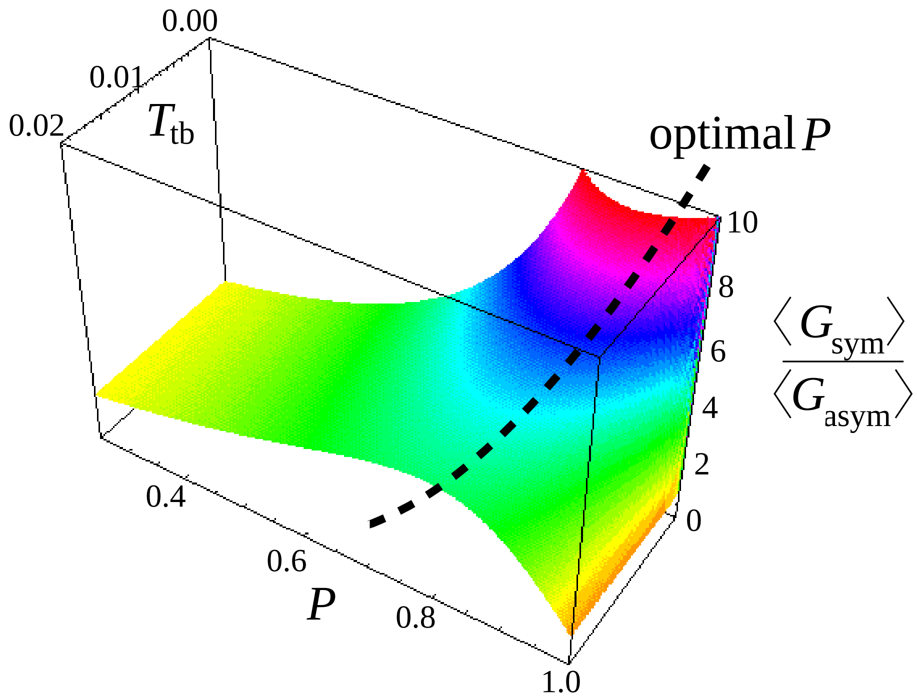
Conductance (two parameters: P & T_b)

- sym. $G_{sym} = \frac{e^2}{h} N \frac{P(1+P)T_b}{(1-P)^2 + 4PT_b}$
- asym. $G_{asym} = \frac{e^2}{h} N \frac{PT_b}{1-P+2PT_b}$

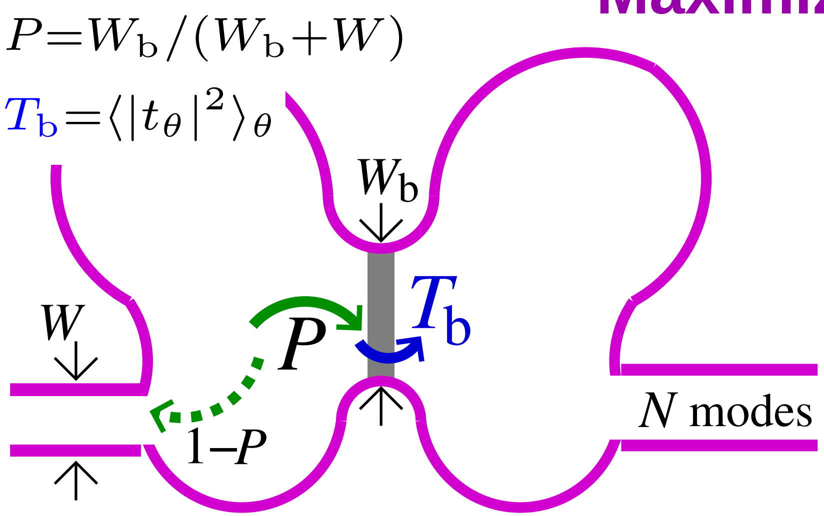
Conductance ratio:

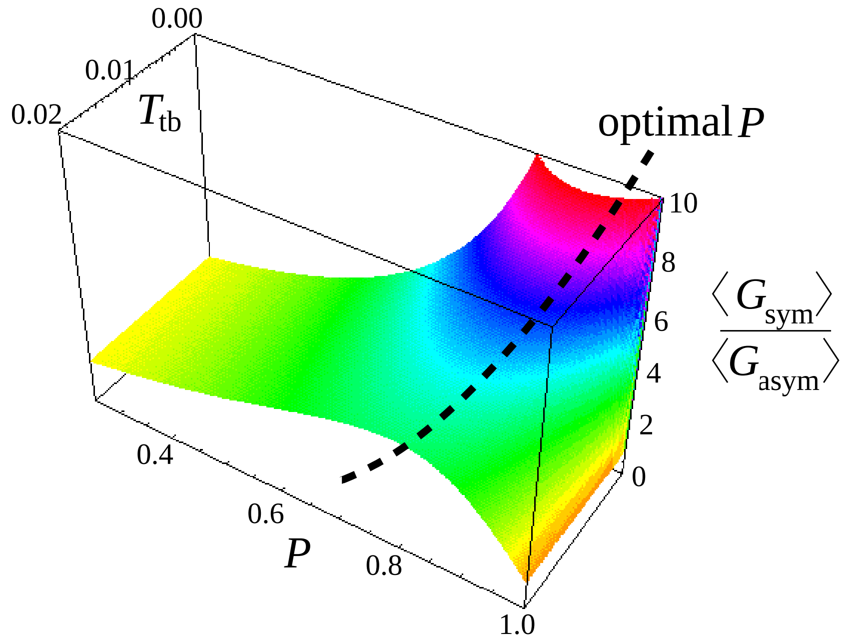
$F_{peak} = G_{sym}/G_{asym}$

- $F_{peak} \gg 1 \Rightarrow$ big peak
- $F_{peak} = 1 \quad \Rightarrow$ no peak



Maximizing the peak





To see a BIG peak:

(1) Tunnelling rate $T_b \to 0$

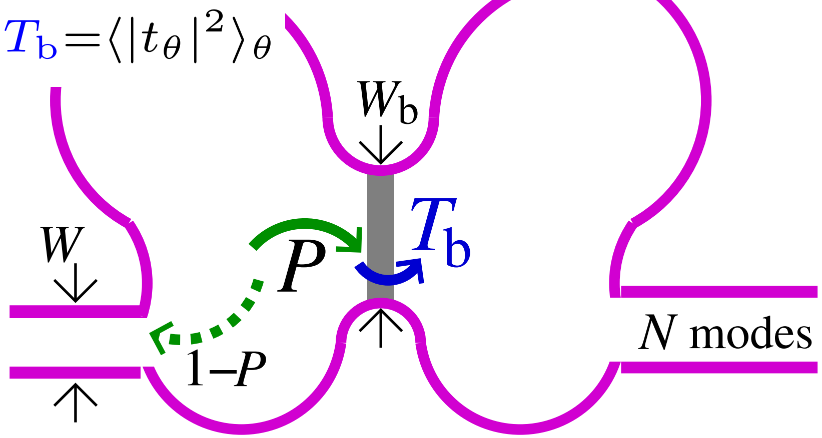
(2) Probability to hit barrier $P \to 1$

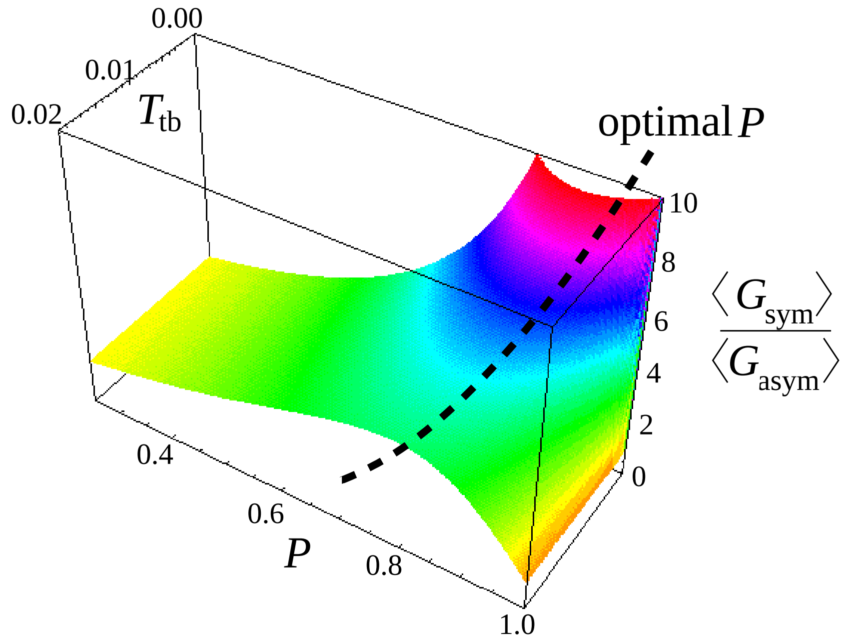
...BUT F_{peak} really maximized by $P = P_{opt} \equiv \frac{1-2T_b^{1/2}}{1-4T_b}$

Maximizing the peak

$P = W_{\rm b}/(W_{\rm b}+W)$

$T_{\rm b} = \langle |t_\theta|^2 \rangle_\theta$





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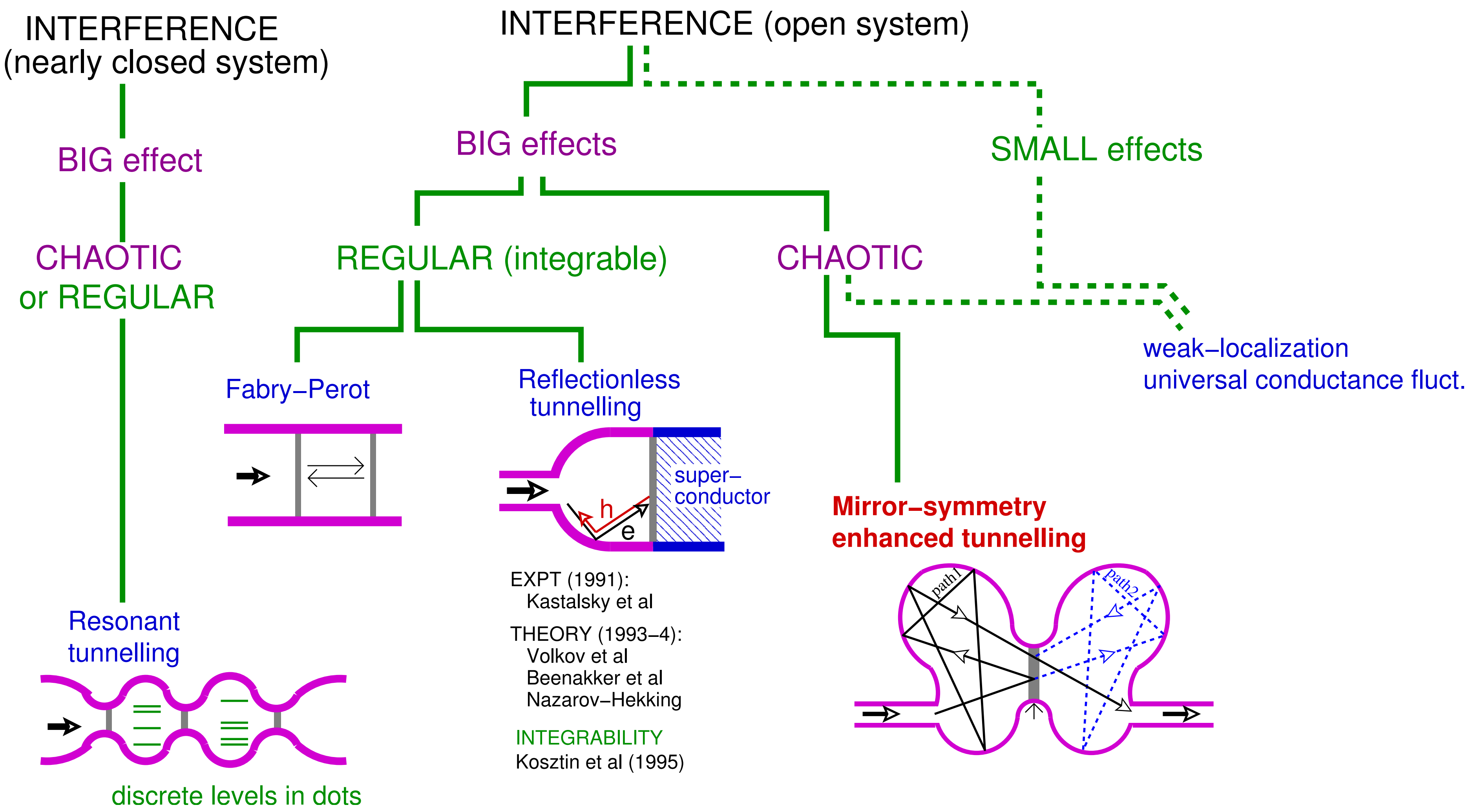
Then for $T_{\rm b} \ll 1$
& $P = P_{\rm opt}$

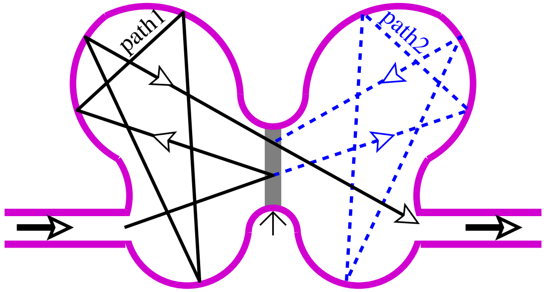
$$\left.\begin{aligned} G_{\rm sym} &= \frac{{\rm e}^2}{h} \times \frac{1}{4} N \\ G_{\rm asym} &= \frac{{\rm e}^2}{h} \times \frac{1}{2} N T_{\rm b}^{1/2} \end{aligned}\right\} F_{\rm peak} \propto \frac{1}{\sqrt{T_{\rm b}}}$$

Family-tree of similar effects

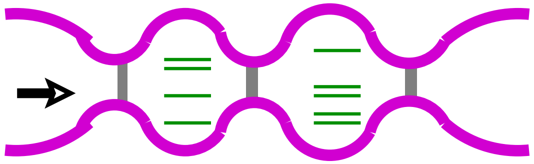


Two Cousins:- resonant tunnel. and reflectionless tunnel.

Mirror−symmetry
enhanced tunnelling

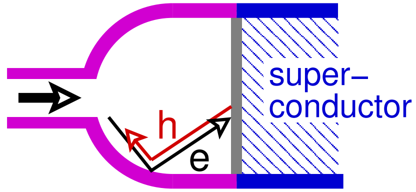


Resonant
tunnelling



discrete levels in dots

Reflectionless
tunnelling

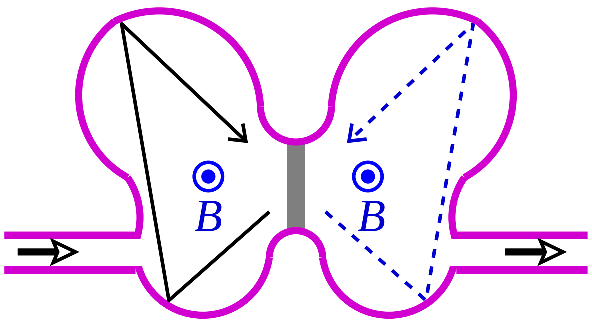


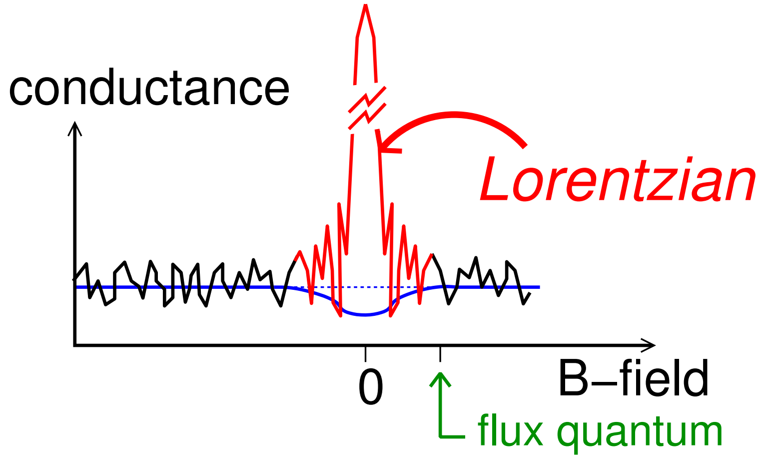
EXPT (1991):
Kastalsky et al

THEORY (1993−4):
Volkov et al
Beenakker et al
Nazarov−Hekking

INTEGRABILITY
Kosztin et al (1995)

Peak-shape with B-field or deformation





Lorentzian width $B = B_c \sqrt{\frac{\tau_0}{\tau_{\mathrm{D}}}}$

approx same width as weak-localization

$B_c \equiv$ one flux quantum

$\tau'_{\mathrm{D}} \sim$ time in double-dot

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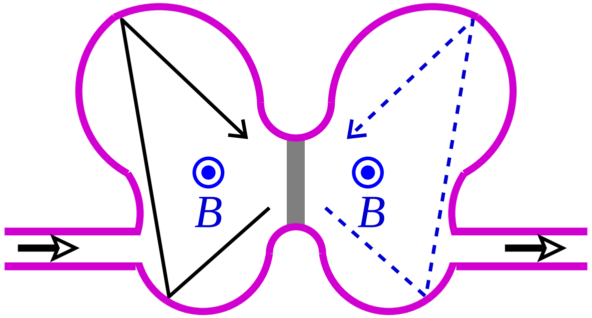

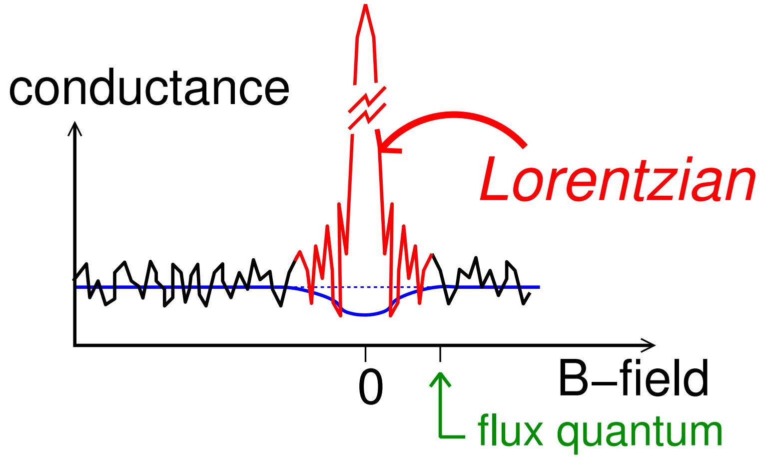

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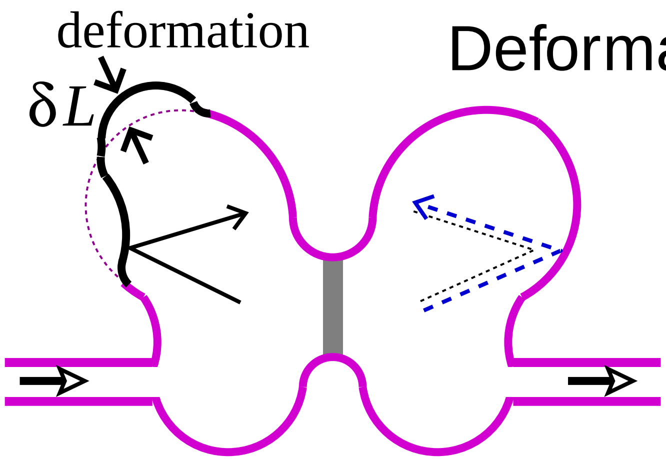
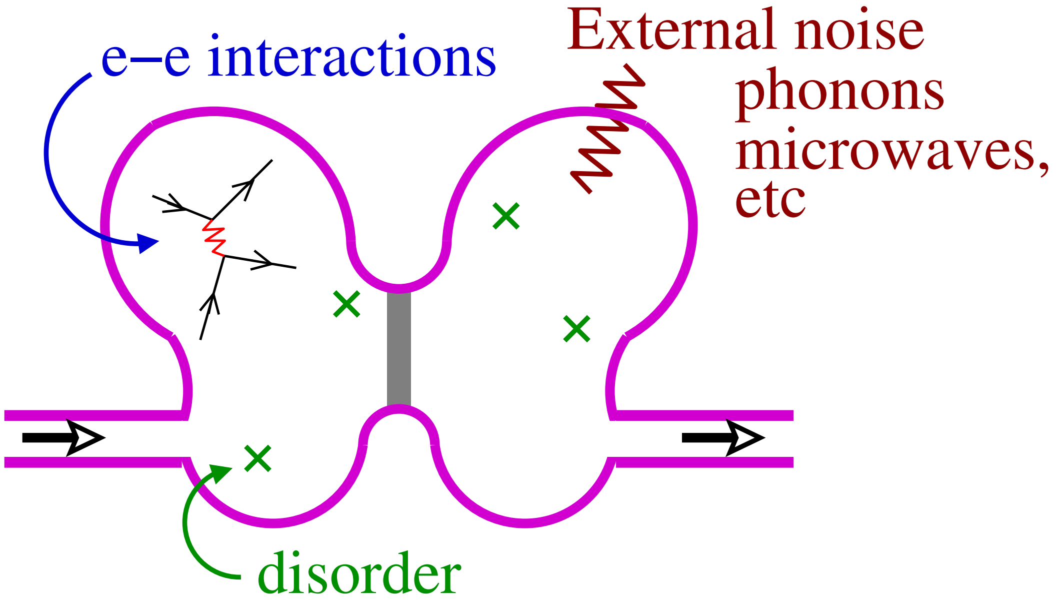


Deformation: *Lorentzian*— width $\delta L \sim \lambda_\mathrm{F} \sqrt{\frac{\tau_0}{\tau_\mathrm{D}}}$

BAD NEWS

no peak *if* deform *small* fraction of λ_F

More BAD NEWS : disorder and dephasing



$$\text{Suppression of peak} = \left[1 + \frac{\tau_D}{\tau_\phi} + \frac{\tau_D}{\tau_{mf}}\right]^{-1}$$

τ_ϕ = dephasing time (as in weak-localization)

τ_{mf} = mean-free path in 2DEG

& $\tau_D \sim$ time electron spends in double-dot

Experimental numbers

Worlds cleanest 2DEG: meanfree path $\sim 500\,\mu$m !!

Fermi wavelength $\sim 50\mathrm{nm}$ Pfeiffer's group (2008)

We choose:

- each dot's diameter $L = 4\,\mu\mathrm{m}$
- barrier tunnelling prob $T_\mathrm{b} = 1.5\times 10^{-3}$ & barrier width ~dot diameter (maximum)

$\Rightarrow$ Maximise peak: $P = 0.93$ so lead's $W = 310\,\mathrm{nm}$ (12 modes)

$$\Rightarrow G_\mathrm{sym} \simeq 3.2\frac{e^2}{h} \qquad G_\mathrm{asym} \simeq 0.22\frac{e^2}{h}$$

Peak is $14\times$background; $F_\mathrm{peak} \simeq 14$

Experimental numbers

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Fermi wavelength $\sim$50nm

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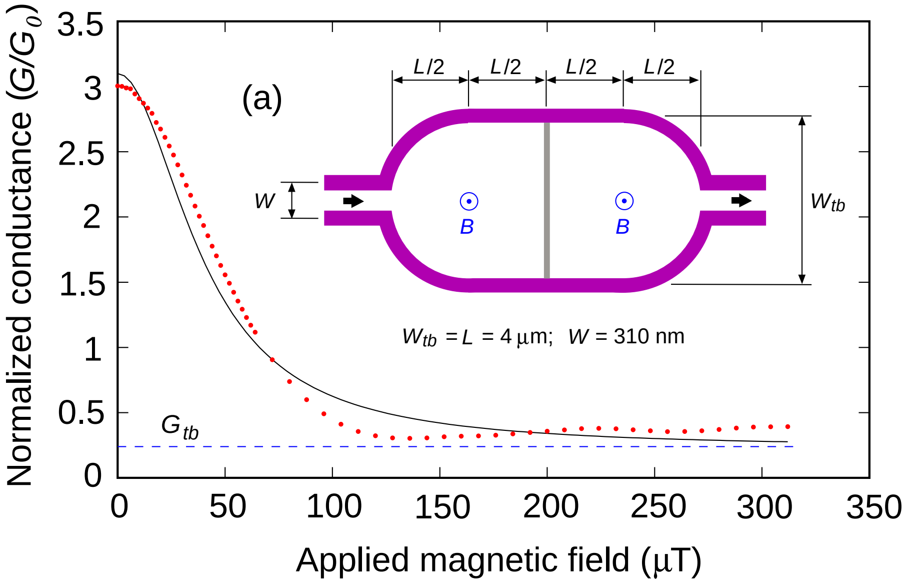
Suppression:

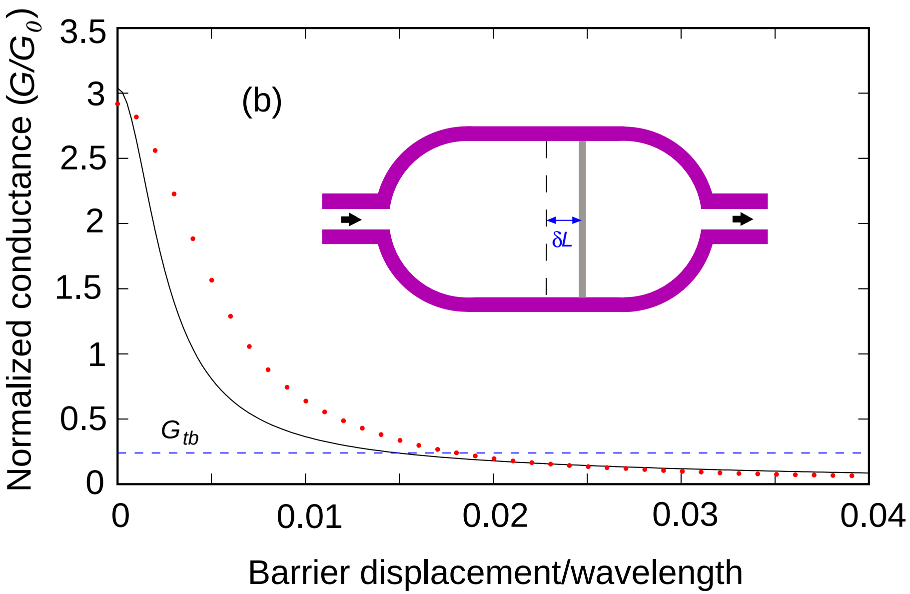
1) asymmetry *irrelevant* if $\delta L < \lambda_{\rm F}/20 \sim 2$nm

2) realistic disorder/dephasing: *reduces* peak to $10\times$background

Detect B-fields: conductance drops by order of magnitude if fifth of a flux-quantum in each dot.

Numerics





Use numbers from experiment
$\Rightarrow$ beyond regime of theory??

Fully solve for waves in double-dot

Recursive Green funct.

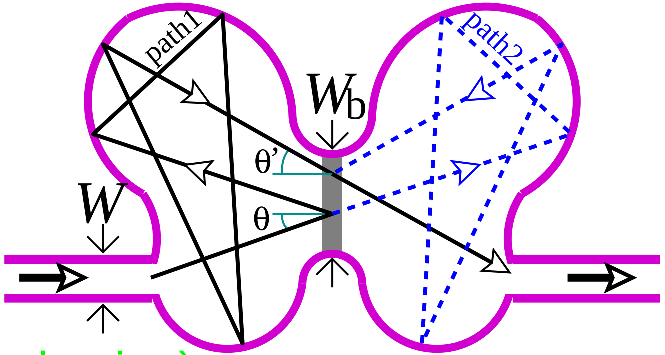
-many vertical slices

no disorder/dephasing

$$G_{\rm sym} = 14 \times G_{\rm asym}$$

B dependence $\sim$ Lorentzian
peak widths $\sim$ theory

Conclusions



Huge new *interference effect*:

barrier can become "invisible"

in symmetric double-dot

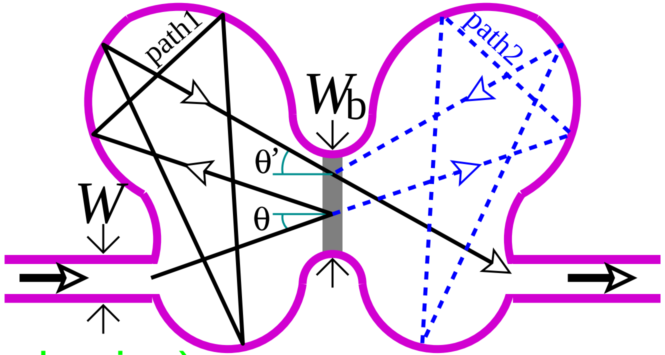
Perfect device (perfect symmetry & no disorder/dephasing)

- peak arbitrarily large

Best "available" device (cleanest 2DEGs & lowest temperatures)

- peak $10\times$ background — it is $> 10\times$ weak-localization dip
- detect less than quantum of flux

Conclusions



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Question for expt: Make two dots symmetric on scale of 2nm ??

Question for theory:

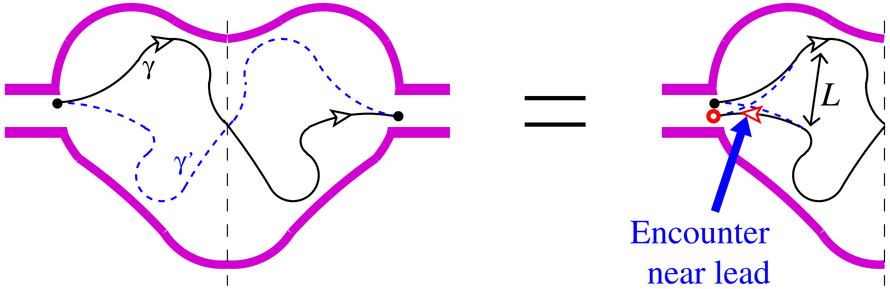
Weak-localization, UCFs, shot noise

Spin-Orbit ??

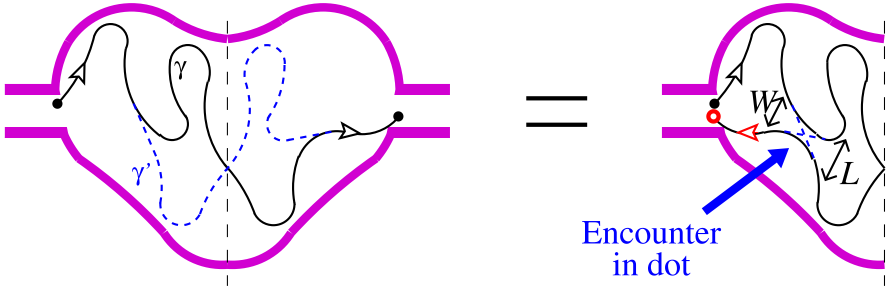
Post-script on weak-loc with mirror-sym.

Baranger-Mello (1996)

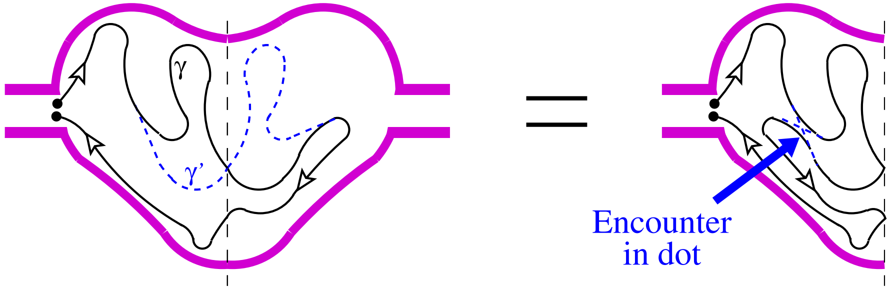
(a) Enhancement of transmission due to left–right symmetry



(b) Reduction of transmission due to left–right symmetry



(c) Reduction of reflection due to left–right symmetry



Conductance peak $= \frac{1}{4} G_0$

destroyed by sym.-breaking

... but B-field independent

Post-script on weak-loc with mirror-sym.

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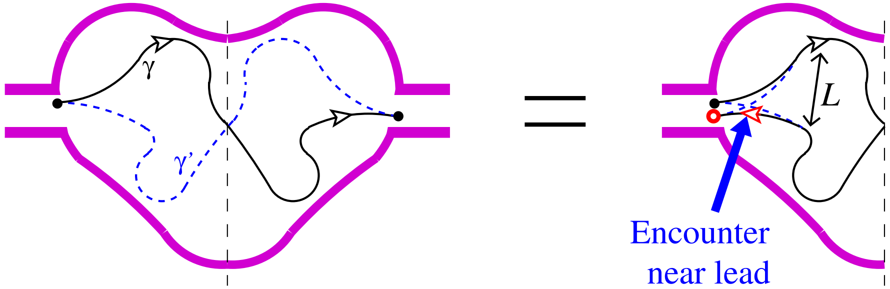

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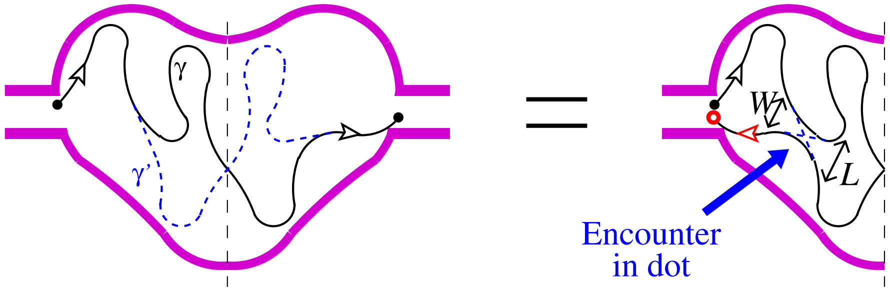
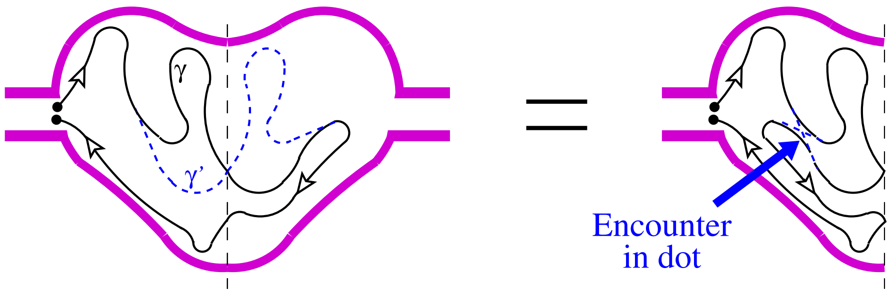
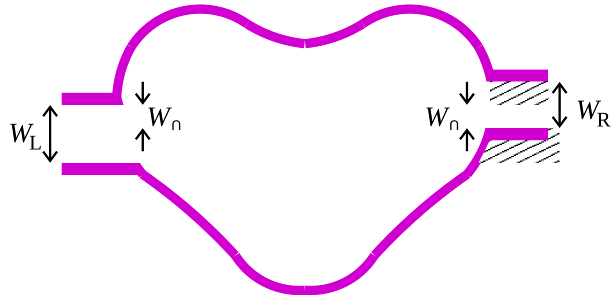


see "shape of conductance peak"

(c) Reduction of reflection due to left–right symmetry





function of lead-displacement

$\propto (1-w)/(1+w)$

w =displacement/width