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# *The best quantum thermoelectric at finite power output*

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Preprint arXiv:1306.0826

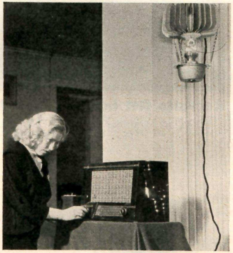
Aachen — Nov 2013

# OVERVIEW

Ioffe (1958)
Inst. Semicond.
Leningrad

***2 Watts***
(80-90 Volts)

$T_{HOT} = 572$ K
$T_{COLD} = 305$ K



**Kerosene Radio** Made in Moscow for use in rural areas, this all-wave radio is reportedly powered by the kerosene lamp hanging above it. A group of thermocouples is heated internally to 570 degrees by the flame. Fins cool the outside to about 90 degrees. The temperature differential generates enough current to operate the low-drain receiver. Regular listeners may want fur-lined union suits, though: It works best in a room with open windows.

www.neazoi.com/technology/thermocouple.htm

MY QUESTION:
What is MAX. efficiency
at GIVEN power output?

♣ Quantum thermoelectric

♣ Nonlinear Landauer-Büttiker

♢ *parasitic* heat flows
— phonons & photons

♢ *inelastic / relaxation*
in quantum system

# CENTRAL RESULTS

Heat-engine efficiency:

$$\eta_{\rm eng} = P/J$$

Output : power $= P = VI$
Input : heat-current $= J$

Refrigerator efficiency
$\equiv$ coeff. of performance (COP):

$$\eta_{\rm fri} = J/P$$

Output : $J$
Input : $P$

[1] ABSOLUTE upper bound on Power Output: $P \leq P_{\rm qb}$

♣ $P_{\rm qb}$ is *quantum-bound* (ill-defined in classical thermodyn)

[2] MAX. efficiency, $\eta_{\rm eng}(P)$, at given power $P$

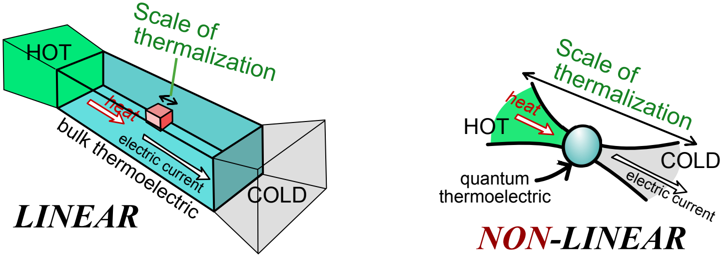
♣ function of $P/P_{\rm qb}$ $\Longleftarrow$ *Quantum*
unlike Carnot efficiency = classical

Example low power : $\eta_{\rm eng}(P) \ \leq \ \eta_{\rm eng}^{\rm Carnot}\left(1 - \alpha_1\sqrt{P/P_{\rm qb}} + \cdots\right)$

# INTRODUCTION

## *Bulk* versus *Quantum* : for LARGE $\Delta T$

Linearity requires temp. drop to be *small on scale of thermalization*




Nonlinear regime : efficiency $\eta$ is meaningful, but $ZT$ is ***NOT***.

Muralidharan-Grifoni (2012), Whitney (2013), Meier-Jacquod (2013), Michelini's poster

Linear formula

$$\eta/\eta_{\mathrm{Carnot}} = \frac{\sqrt{ZT+1}-1}{\sqrt{ZT+1}+1}$$

Dictionary (for linear people)

$$ZT = \infty \iff \eta = \eta_{\mathrm{Carnot}}$$

$$ZT = 3 \iff \eta = \tfrac{1}{3}\eta_{\mathrm{Carnot}}$$

$$ZT = 0 \iff \eta = 0$$

## METHOD: scattering theory beyond linear response

Heat current: $$J_L = \int_{-\infty}^{\infty} \frac{\mathrm{d}\epsilon}{h}\, \epsilon\, \mathcal{T}_{RL}(\epsilon) \left( f\left[\frac{\epsilon - eV_L}{k_\mathrm{B}T_L}\right] - f\left[\frac{\epsilon - eV_R}{k_\mathrm{B}T_R}\right] \right)$$

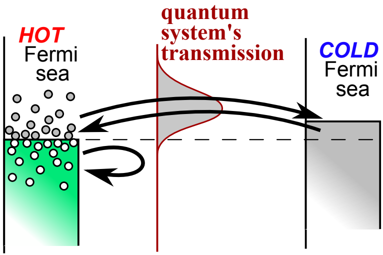
- ♢ transmission function $\mathcal{T}_{RL}(\epsilon) = \mathrm{tr}\big[\mathcal{S}^\dagger(\epsilon)\, \mathcal{S}(\epsilon)\big]$
- ♢ Fermi-Dirac for ingoing particles: $f\big[(\epsilon - eV_j)/k_\mathrm{B}T_j\big]$
- ♣ obeys 2nd law thermodyn, etc. Whitney PRB (2013)

beyond linear-response : Hartree-like interactions – *self-consistent*

Christen-Büttiker (1996)

Self-consistent loop: $\mathcal{S}(\epsilon)$ ⇄ potential-distrib. in system

# ORIGIN of THERMOELECTRICITY




$$I = 0$$
$$V = V_{\text{stop}}$$
$$P = IV = 0$$

Mahan,Sofo (1996). Humphrey,Linke (2005)

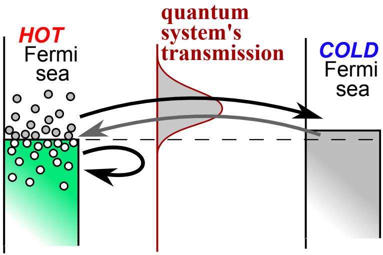
Vanishing transmission width

$\Rightarrow$ reversibility (no entropy generated)

$\Rightarrow$ Carnot efficiency $\eta = \eta_{\text{carnot}} = 1 - T_R/T_L$

... but *no* power

# ORIGIN of THERMOELECTRICITY




$$I \neq 0$$
$$V \neq 0$$
$$P = IV = max.$$

Efficiency at max. power

- vanishing transmission width;

  Esposito,Lindenberg,van den Broeck (2009) $\Rightarrow$ Curzon Ahlborn efficiency

  Curzon, Ahlborn (1975), Novikov (1957), Chambadal (1957)

- non-vanishing transmission width:

  Nakpathomkun, Xu, Linke (2010), Leijnse, Wegewijs, Flenberg (2010)
  Hershfield, Muttalib, Nartowt (2013), ...Others

  ... *higher* max power but *lower* efficiency at that power

## ANSWERING MY QUESTON

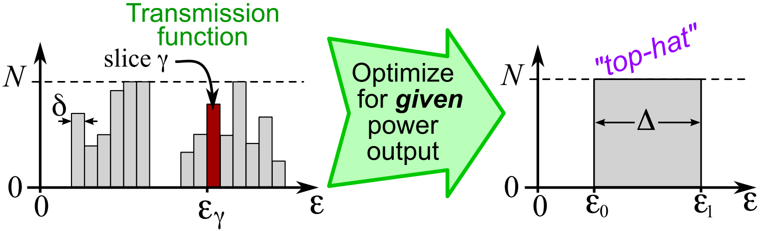
What is MAXIMUM EFFICIENCY for **GIVEN** power output?

# OPTIMIZING EFFICIENCY for GIVEN POWER OUTPUT

$(n+1)$ variables: $n$ slices + bias, $V$

one constraint : power $= P$

♣ *want* minimum heat-flow $J$ for given $P$



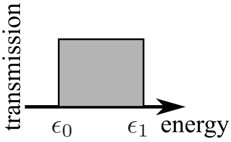

Proof: changing height $\tau_\gamma$ of slice $\gamma$, decreases $J$ (increases efficiency)

***if*** $$0 > \left.\frac{\partial J}{\partial \tau_\gamma}\right|_P = \left(\frac{\epsilon_\gamma}{eV} - \frac{J'}{P'}\right) \times \left.\frac{\partial P}{\partial \tau_\gamma}\right|_V$$

primed = $\mathrm{d}/\mathrm{d}V$

# TRANSCENDENTAL EQUATION

For given temperatures $T_L$ (hot) & $T_R$ (cold)




$$\epsilon_0 = \frac{eV}{1 - T_R/T_L} \qquad \epsilon_1 = eV\,\frac{J'}{P'}$$

primed = $\mathrm{d}/\mathrm{d}V$

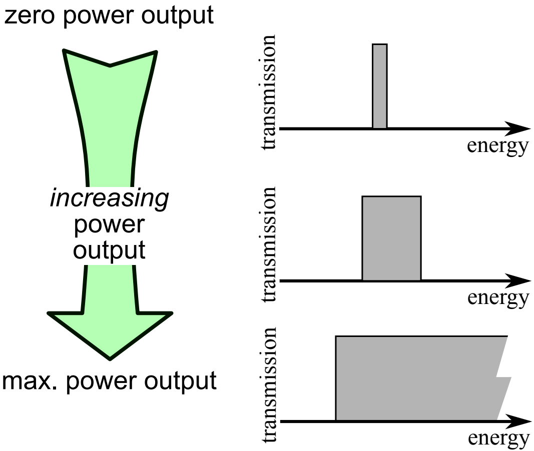
Energy-integrals in $J$ and $P$ are Fermi-functions $\times$ top-hat

$P$ & $J$ are sums of *logs* and dilog.-functions

$$\ln\left[1 + \mathrm{e}^{-(\epsilon - eV_j)/k_B T_j}\right] \;\&\; \mathrm{Li}_2\left[-\mathrm{e}^{-(\epsilon - eV_j)/k_B T_j}\right]$$

Get $\epsilon_1$ from above *transcendental eq.* for given $T_L$ (hot) & $T_R$ (cold)

# OPTIMAL TOP-HAT WIDTH



## UPPER-BOUND on POWER OUTPUT for $N$ transverse modes

Refrigerator cooling power: $J \;\leq\; \frac{1}{2} J_{\rm qb} \;\equiv\; \frac{\pi^2}{12h} N (k_{\rm B} T_L)^2$

Heat-engine electrical power:

$$P \;\leq\; P_{\rm qb} \;\equiv\; \frac{A_0 \pi^2}{6h} N \left(k_{\rm B} T_L - k_{\rm B} T_R\right)^2$$

with $A_0 \simeq 0.192$

- Purely **quantum**, i.e. irrelevant for $N \to \infty$
- $J_{\rm qb}$ = Pendry (1983) as limit on entropy flow
  $\Longrightarrow$ "single mode fermionic" analogue of black-body
- Large $N$ — Bjorn Sothmann's talk Jordan et al (2013)

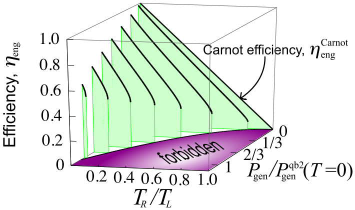
For Ioffe's "Kerosene Radio" set-up :

$J_{\rm qb}, P_{\rm qb} \sim$ 10nW per transverse mode

$\Rightarrow$ 100W *needs cross-section 1cm×1cm*

# MAX. EFFICIENCY for GIVEN POWER OUTPUT

Maximal heat-engine efficiency

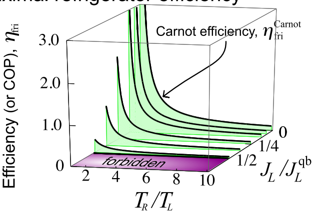



Heat-eng. : *small* power output, $P$.

$$\eta = \eta^{\rm Carnot}\left(1-\alpha_1\sqrt{\frac{P}{P_{\rm qb}}}+\cdots\right)$$

where $P_{\rm qb}$ is upper-bound

Maximal refrigerator efficiency




Fridge : *small* power output, $J$.

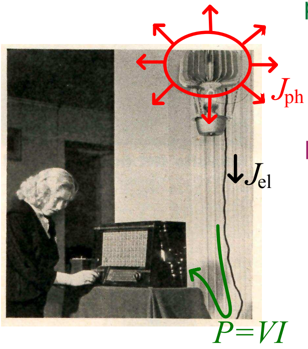
$$\eta = \eta^{\rm Carnot}\left(1-\alpha_2\sqrt{\frac{J}{J_{\rm qb}}}+\cdots\right)$$

where $J_{\rm qb}$ is upper-bound

# PARASITIC PHONON/PHOTON FLOWS

Black-body photons: $J_{\rm ph} \propto (T_{\rm hot}^4 - T_{\rm cold}^4)$

phonon in nanostructures Heron et al (2009-11)




Total efficiency $\eta_{\rm el\&ph} = \dfrac{P}{J_{\rm el} + J_{\rm ph}}$

Max efficiency - don't care what $P$

(not given $P$)

- *No phonons/photons:*
  max $\eta_{\rm el\&ph}$ = NARROW transmission
  $\Longrightarrow \eta_{\rm el\&ph} \to \eta_{\rm Carnot}$
- *Phonons/photons dominate:*
  max $\eta_{\rm el\&ph} \Leftrightarrow$ max $P$
  = WIDE transmission

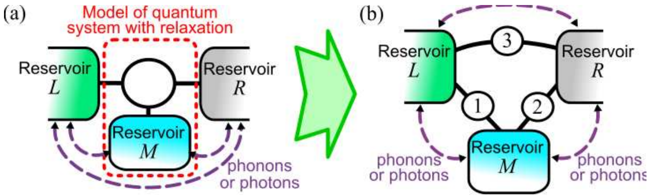
Max efficiency at given $P$ :

$$\eta_{\rm el\&ph}(P) = \frac{P\,\eta_{\rm el}(P)}{P + \eta_{\rm el}(P) J_{\rm ph}}$$

## **RELAXATION** modelled as Buttiker "voltage probe" (1988)

Sometime – inelastic scattering may *increase* efficiency

see Casati's talk & Ora's talk




## REALLY HARD PROBLEM !!

*ANSWERED* in 2 limits : *low* power and *max* power

Over-estimate *never* exceed results *without* relaxation.

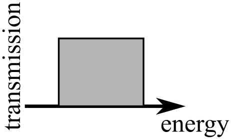
... *open question* for intermediate powers

# CONCLUSIONS

♣ Max. efficiency at *given* power : "top-hat"

transcendental eq. for position/width

⇒ *Width grows with power*




♣ Results : [1] max. possible power *(quantum)*

[2] max. possible efficiency *(quantum)*

Is this the *BEST* thermoelectric at finite power output??

♣ *Open question:* relaxation at intermediate power outputs?

♣ *Open question:* strongly correlated systems (Kondo, Luttinger, etc)*?*

♢ *How to make top-hat?*

Buttiker said "top-hat = band = chain quantum-dots"